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Stochastic Porous Media Equations

Springer

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Preface

This book is concerned with stochastic porous media equations with main emphasis on existence theory, asymptotic behaviour and ergodic properties of the associated transition semigroup. The general form of the porous media equation is

$$dX - \Delta\beta(X)dt = \sigma(X)dW, \tag{1}$$

where $\beta : \mathbb{R} \to \mathbb{R}$ is a monotonically increasing function (possibly multivalued) and W is a cylindrical Wiener process.

Stochastic perturbations of the form $\sigma(X)\dot{W}$ in stochastic porous media equation were already considered by physicists but until recently no rigorous mathematical existence result was known. In specific models the noise arises from physical fluctuations of the media which in a first approximation can be taken of the form $(a + bX)\dot{W}$.

The porous media equation driven by a Gaussian noise, besides their relevance in the mathematical description of nonlinear diffusion dynamics perturbed by noise, has an intrinsic mathematical interest as a highly nonlinear partial differential equation, which is not well posed in standard spaces of regular functions. In fact the basic functional space for studying this equation is the distributional Sobolev space H^{-1} and this is due to the fact that the porous media operator $y \to -\Delta\beta(y)$ is m-accretive in the spaces H^{-1} and L^1 only. Since the Hilbertian structure of the space is essential for getting energetic estimates via Itô's formula, H^{-1} was chosen as an appropriate space for this equation.

Compared with the deterministic porous media equation which benefits from the theory of nonlinear semigroups of contractions in both the spaces L^1 and H^{-1}, the existence theory of the corresponding stochastic equations is not a direct consequence of general theory of the nonlinear Cauchy problem in Banach spaces. In fact, a nonlinear stochastic equation with additive noise (or with special linear noise) is formally equivalent with a nonlinear random differential equation with non-smooth time-dependent coefficients, which precludes the use of standard existence result for the deterministic Cauchy problem. However, the existence theory for

stochastic infinite dimensional equations uses many techniques of nonlinear Cauchy problems associated with deterministic m-accretive nonlinear operators.

This book is organized into seven chapters. Chapter 1 is devoted to some standard topics from stochastic and nonlinear analysis mainly included without proof because they represent a necessary basic background for the subsequent topics.

Chapter 2 is devoted to existence theory for stochastic porous media equations with Lipschitz nonlinearity and may also be viewed as a background for the theory developed in Chap. 3, which is the core of the book. This chapter treats existence theory for equations with maximal monotone nonlinearities which have at most polynomial growths. The principal model described by this class of equations is the slow and fast diffusion processes. Besides existence, the extinction in finite time for fast diffusions and finite speed of propagation for slow diffusions are also studied.

Chapter 4 is devoted to the so-called variational approach to stochastic porous media equations. In a few words, the idea is to represent the equation as an infinite dimensional stochastic equation associated with a monotone and demi-continuous operator from a reflexive Banach space V to its dual V' and apply the standard existence theory developed in the early 1970s by E. Pardoux, N. Krylov and B. Rozovskii.

Chapter 5 is devoted to stochastic porous media equations with nonpolynomial growth to $\pm\infty$, for the diffusivity β, a situation which was excluded from the previous H^{-1} approach and which uses an L^1 treatment based on weak compactness arguments. The solution obtained in this way is weaker than in the previous case but applies to a larger class of functions β.

Chapter 6 is concerned with stochastic porous media equations in the whole $\mathbb{R}^d$.

Chapter 7 is devoted to existence and uniqueness of invariant measures for the transition semigroup associated with stochastic porous media equations.

These lecture notes have grown out of joint works and collaborations of authors in the last decade. They were written during their visits to Scuola Normale Superiore di Pisa and Bielefeld University.

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Contents