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Asymptotic Integration of Differential and Difference Equations

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Dedicated to

Roy

and

Margaret

Preface

This book is concerned with the asymptotic behavior for solutions of certain classes of linear differential equations as well as corresponding results for solutions of linear difference equations. The type of asymptotic analysis we employ is based on some fundamental principles attributed to Levinson [100]. While he applied them to a special class of differential equations, subsequent work has shown that the same principles lead to asymptotic results for much wider classes of differential and also difference equations. For differential equations, his approach, which is now referred to as “asymptotic integration,” was utilized in an excellent monograph by Eastham [53] and covers many results up through the mid-1980s. We extend the usage of “asymptotic integration” to describe similar results for difference equations.

Our purpose is twofold: First, in Chaps. 2–7, broad types of theoretical results will be discussed which apply to rather general classes of systems. In the case of differential equations, these extend and supplement Eastham’s work by including many newer contributions and some earlier results not found there. For difference equations, a parallel approach yields a similar bank of general results and extends a much earlier treatment by Rapoport [131]. Second, we consider in Chaps. 8 and 9 applications to some special situations, usually scalar equations with coefficients having particular properties, and compare what can be achieved by applying results from the general approach with what was previously known using ad hoc methods.

The idea for developing asymptotic representation results for both differential and difference equations in parallel came from a series of lectures in 2000 by the second author at Kumamoto University, Japan. This was supported by a grant from the Japanese government and hosted by Prof. M. Kohnno. In 2002, we began collaborating on several research projects which followed this pattern of parallel development. Then while on sabbatical in Provence, we decided to begin assembling our results and others from many authors which either directly or indirectly apply Levinson’s principles and which can be treated from the same perspective. A final push to complete work on this book came in 2013 while the first author was on sabbatical in Olomouc, Czech Republic, and we were able to work together again for several months, especially concentrating on applying the general results

to special classes of scalar equations. We thank Profs. J. Andres and D. Livingstone at Palackého University for their support during this time.

We acknowledge the contributions of many authors whose work is cited in this book as well as other colleagues or collaborators who have influenced our mathematical development. In particular, we thank our Doktorvaeter, R.J. Sacker (University of Southern California), and W.B. Jurkat (Syracuse and Ulm) for introducing us to differential equations and giving us an appreciation of and respect for research. Also, we remember W.A. Harris, Jr., who participated in some of the earlier developments.

Finally, we thank our spouses, Roy and Margaret, for their encouragement and support during the long process of writing this book as well as their understanding for the time taken away from family.

Tacoma, WA, USA
San Diego, CA, USA
February 2015

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Donald A. Lutz

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