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Roman Trobec · Gregor Kosec

Parallel Scientific Computing

Theory, Algorithms, and Applications
of Mesh Based and Meshless Methods

 Springer

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ISSN 2191-5768 ISSN 2191-5776 (electronic)
SpringerBriefs in Computer Science
ISBN 978-3-319-17072-5 ISBN 978-3-319-17073-2 (eBook)
DOI 10.1007/978-3-319-17073-2

Library of Congress Control Number: 2015934954

Springer Cham Heidelberg New York Dordrecht London

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Printed on acid-free paper

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(www.springer.com)

To all who make our lives worthwhile.

Preface

The scientific computing and computer simulations on modern, powerful computers are tools that can reduce the costs of developing new machines, evaluate different environmental risks, simulate the evolution of different natural or technological phenomena, and conduct virtual experiments that are too dangerous or impossible to perform in laboratories, amongst many other possibilities.

This book is concentrated on the synergy between computer science and numerical analysis. It is written to provide a firm understanding of the described approaches to computer scientists, engineers or other experts who have to solve real problems. The meshless solution approach is described in more detail, with a description of the required algorithms and the methods that are needed for the design of an efficient computer program. Most of the details are demonstrated on solutions of practical problems, from basic to more complicated ones. We believe that this book will be a useful tool for any reader interested in solving complex problems in real computational domains.

We are grateful to all our colleagues who have contributed to this book through discussions or by reading the material, in particular to Marjan Šterk and Božidar Šarler who initiated and supported the research on meshless methods in our research community. Many thanks to Monika Kapus-Kolar and Matjaž Depolli, who carefully read our text and resolved many formal and linguistic inconsistencies. We are indebted to the Jožef Stefan Institute and the Slovenian Research Agency for their support of our work.

Ljubljana, February 2015

Roman Trobec
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