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Many mathematicians from all over the world have been involved in a way or another in C.I.M.E.'s activities over the years. The main purpose and mode of functioning of the Centre may be summarised as follows: every year, during the summer, sessions on different themes from pure and applied mathematics are offered by application to mathematicians from all countries. A Session is generally based on three or four main courses given by specialists of international renown, plus a certain number of seminars, and is held in an attractive rural location in Italy.

The aim of a C.I.M.E. session is to bring to the attention of younger researchers the origins, development, and perspectives of some very active branch of mathematical research. The topics of the courses are generally of international resonance. The full immersion atmosphere of the courses and the daily exchange among participants are thus an initiation to international collaboration in mathematical research.

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P.R. Kumar • Martin J. Wainwright •
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Mathematical Foundations of Complex Networked Information Systems

Politecnico di Torino, Verrès, Italy 2009

Fabio Fagnani, Sophie M. Fosson, Chiara Ravazzi
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Preface

In the past 20 years, we have been witnessing a revolution in information and communication technologies, consisting in an astonishing progress in different research fields. Innovations have been developed not only in the classical fields of wireless communication (design of high performance codes very close to the Shannon limit), computer engineering (parallel computing, computer vision, data mining), bioinformatics (genome interpretation), and VLSI technologies, but also in new areas such as wireless sensor networks and smart dust, air traffic control, large scale surveillance, unmanned mobile multi-vehicle systems, biological networks interpretation, and so on.

All of these problems deal with complex systems and large scale optimization, and require a prohibitive amount of computational effort even in low dimensions. The complexity calls for robust and adaptable solutions. In spite of the inherent difficulties, fundamental contributions have already appeared, based on the principle that most of complex systems are the result of the interactions of numerous, but rather simple entities. The complexity is then a consequence of the interactions architecture, that can be described by a network, and solutions can be developed from mathematical models that describe how the local rules originate the global behavior.

The mathematical tools useful to address these new scientific issues come mainly from combinatorics, probability theory, and statistical mechanics. The concept of graph plays a prominent role: not only it is the natural model for any communication network, but also it describes interactions among variables, as exploited, e.g., in graphical models in modern coding theory.

In particular, random graphs are a suitable model to describe complex networks (e.g., internet, wireless communication networks, sensor networks). Randomness enters at different levels, to mimic the development of complex networks, to model faults or noise in the communication links, or also to find typical codes or algorithms with good properties. For a deep analysis of such random models, probabilistic techniques are essential, among which we mention concentration inequalities, large deviation theory, and percolation. In case of aggregation of simple entities, where

complexity rises from the network of interactions, statistical mechanics is a natural approach.

The summer school “Mathematical Foundations of Complex Networked Information Systems” was organized by CIME, with the contribution of Newcom++, in Verrès (Aosta Valley, Italy) in 2009. The aim of the school was to give an introduction to some of the fundamental scientific issues emerging in these disciplines. It consisted of four courses, each of 6 h.

The course of Prof. Béla Bollobás focused on random graphs: it provided all the necessary probabilistic tools and proposed a number of different ways to model randomness.

The course of Prof. P.R. Kumar focused on communication networks, in particular on the issues of the transmission of information along a network where there are simultaneously many potential receivers and many potential transmitters. Prof. Kumar presented the basic information theoretic aspects of wireless communication networks.

The other two courses addressed the analysis of distributed algorithms over networks and, more generally, over graphical models. Particular emphasis was given to the famous message passing algorithms which have applications in a broad variety of contexts including artificial intelligence, distributed inferential statistics, coding theory, and combinatorial optimization. The course of Prof. Riccardo Zecchina described the statistical physics interpretation of these algorithms with special attention to applications in classical combinatorial optimization problems. The course by Prof. Martin J. Wainwright instead focused on the probabilistic aspects of message passing algorithms and their relation with the relaxation techniques in optimization theory.

About 70 people from all over the world attended the school. Among them, there were senior graduate students and post-doc researchers in pure and applied mathematics, information engineering, and physics.

The overall aim of this book is to record the results presented during the summer school.

Torino, Italy
Torino, Italy
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