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Geometric Invariant Theory for Polarized Curves

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Preface

Geometric invariant theory (or GIT for short), as developed by D. Mumford in the 1960s (using ideas of D. Hilbert in classical invariant theory), studies linearized actions of (reductive) linear algebraic groups on (quasi-projective) algebraic varieties and it provides techniques for constructing a categorical quotient within the category of algebraic varieties. One of the most successful applications of GIT is the construction of moduli spaces. This is usually achieved by first constructing a variety that parametrizes the desired geometric objects suitably rigidified (e.g., varieties embedded in projective spaces, cycles in projective spaces, coherent sheaves together with a surjection from a given coherent sheaf, etc.) in such a way that the relation of forgetting the rigidification is given by the action of a linear algebraic group; then, the GIT quotient provides the desired moduli space.

In recent years, the interest in GIT has been reinvigorated by its connections with the Minimal Model Program (MMP). One expects that the minimal model of a moduli space is a new moduli space that is related to the original one via a sequence of birational transformations which should as well admit a modular description. GIT provides a natural framework to find new birational models of a given moduli space by varying either the linearization of the action (as in the theory of Variations of GIT) or the initial rigidification of the moduli problem. This strategy has been recently carried out for the first steps of the MMP for the moduli space of Deligne–Mumford's stable curves.

In this work, we investigate the GIT quotients of the Hilbert and Chow schemes of curves of degree d and genus $g \geq 2$ in projective space of dimension $d - g$, as the ratio $v := \dfrac{d}{2g-2}$ decreases.

We prove that the first three values of v at which the GIT quotients change are given by $v = 4, 3.5$ and 2. We show that, for $v > 4$, L. Caporaso's results hold true for both the Hilbert and Chow semistable loci, which map to the moduli stack of Deligne–Mumford's stable curves. If $3.5 < v < 4$, the Hilbert and Chow semistable loci coincide and they map to the (non-separated) moduli stack of Hyeon–Morrison's weakly pseudo-stable curves. If $2 < v < 3.5$, the Hilbert

and Chow semistable loci coincide and they map to the moduli stack of Schubert's pseudo-stable curves. We also analyze in detail the critical values $v = 3.5$ and $v = 4$, where the Hilbert semistable locus is strictly smaller than the Chow semistable locus.

As an application of our results, we get two new compactifications of the universal Jacobian over the moduli stack of weakly pseudo-stable curves and of pseudo-stable curves, which provide two modular birational models of Caporaso's compactification of the universal Jacobian over the moduli stack of stable curves.

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