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Bifurcation without Parameters

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Preface

This monograph is devoted to the study of dynamical systems with manifolds of equilibria near points at which normal hyperbolicity of these manifolds is violated. It is targeted at mathematicians with at least basic familiarity with classical bifurcation theory. Although methods and concepts are briefly introduced, prior knowledge of center-manifold reductions and normal-form calculations may help to appreciate the presentation.

Manifolds of equilibria arise frequently in parameter dependent systems—by continuation of a trivial equilibrium. Loss of hyperbolicity of such equilibria yields qualitative changes of the local dynamics. Its study is one of the main objectives of classical bifurcation theory.

Here, however, we are interested in manifolds of equilibria that are not caused by additional parameters. Still, qualitative changes of the local dynamics close to the manifold of equilibria occur at points at which normal hyperbolicity of these manifolds breaks down. To exclude not only given but also any unknown or "hidden" parameters, we require the absence of any flow-invariant foliation transverse to the manifold of equilibria at the bifurcation point. We call the emerging theory *bifurcation without parameters*.

On first glance our setting appears to be very degenerate. Indeed, vector fields with manifolds of equilibria form a set of infinite codimension in the space of all smooth vector fields. However, there is a surprisingly rich and diverse collection of applications ranging from networks of coupled oscillators, viscous and inviscid profiles of stiff hyperbolic balance laws, standing waves in fluids, binary oscillations in numerical discretizations, population dynamics, memristor circuits, cosmological models, and many more.

Note that parameter dependent systems, likewise, form a set of infinite codimension in the space of vector fields with manifolds of equilibria—if we consider the parameters as fixed phase variables. As classical bifurcation theory is justified by its applicability, so is bifurcation theory without parameters.

This monograph is a slightly extended version of my Habilitation thesis at the Free University Berlin in 2013. I am much indebted to the Free University Berlin in

general and to Prof. Dr. Bernold Fiedler in particular for providing an environment where research and teaching, alike, is not only successful but also very enjoyable.

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