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Inverse M-Matrices and Ultrametric Matrices

Springer

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To our families

Preface

In this monograph we collect our work over the past 20 years linking ultrametric matrices with potential theory of finite state Markov chains and their consequences for the inverse M-matrix problem. This was triggered when Martínez, Michon and San Martín [44] proved that ultrametric matrices are inverse M-matrices. Nabben and Varga [51] provided a linear algebra proof of this fact, spreading ultrametricity towards linear algebra. Further developments were stated with the introduction of generalized ultrametric matrices by McDonald, Neumann, Schneider and Tsatsomeros [47] and Nabben and Varga [52].

Our presentation is grounded in a conceptual framework in which potential equilibrium and filtered matrices play a fundamental role. One of our main focal points is the study of the graph of connections associated with ultrametric and generalized ultrametric matrices. A fruitful line of research is to exploit the tree structure underlying these matrices, which provides a tool for understanding the associated Markov chain (see our papers [20] and [22]).

As it turns out, potential matrices perform well under Hadamard operations. We deal with an invariance of potentials under some Hadamard functions. Most notably powers and exponentials, see Neumann in [54], Chen in [11, 12] and our papers [23, 25].

This book is not intended to contain a complete discussion of the theory of inverse M-matrices and primarily reflects the interests of the authors in key aspects of this theory.

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