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Singular Perturbations

Introduction to System Order Reduction Methods with Applications

Springer

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We dedicate this book to the memory of our friend and colleague Alexei Pokrovskii (1948–2010)

Foreword

This book is a tribute to Alexei Pokrovskii (1948–2010) who introduced the Russian and Irish coauthors and who generously promoted and made important contributions to our understanding of singular perturbations from an applied and geometric perspective.

The three authors continue and explain many recent results on the asymptotics of slow integral (or invariant) manifolds and their stability. They do so by cleverly describing a series of illustrative examples of increasing complexity and reality. Many applications in chemical kinetics are particularly impressive, as is their ultimate study of two-dimensional canards and higher-dimensional black swans.

Readers of this clear and well-motivated monograph will be prepared to advance to the even more sophisticated research literature that takes a dynamical systems approach to multi scale systems. The authors are to be congratulated on completing a tough job, very well done. They've certainly earned our appreciation and that of future students.

Seattle, WA
December 2013

Robert O'Malley

Preface

The idea of using a small parameter to set up a perturbation series has been with us since at least the work of Stokes in 1847[1] on the investigation of water waves. The use of integral manifolds, with a small parameter, is of more recent vintage. It can be found in [45, 72, 75, 92, 114, 170, 197, 217, 218]. Over the past 50 years there have been many books devoted to regular and singular perturbations, but there are few books in which singular perturbations are combined with integral manifolds. Moreover, many of these were published only in Russian. The purpose of the present book is to fill this gap.

We deal with a system of first order ODEs some of which are singularly perturbed, i.e., when the small parameter is set to zero the ability to satisfy all initial conditions is lost. We introduce a method for the qualitative analysis of these singularly perturbed ODEs. The method relies on the theory of integral manifolds, which essentially replaces the original system by another system on an integral manifold of lower dimension. The lowering of the dimension occurs due to the decomposition of the original system in the vicinity of the integral surface into the independent "slow" subsystem and the "fast" subsystem. If the slow integral manifold is attracting, then the analysis of the original system can be replaced by the analysis of the slow subsystem. In the language of perturbation theory a slow integral manifold is associated with the outer (slow) solution and a fast integral manifold is associated with boundary layer (fast) corrections.

The book proceeds with the interplay of theory and illustrative examples, in many cases taken from physical problems. There are many such examples in Chap. 3, where the reader is introduced at an easy pace to the use of the theory. As the chapters progress, the theory and corresponding examples become more sophisticated. In Chaps. 7 and 8 we deal with systems where the usual hypotheses in integral manifold theory are violated. The method of solution is then illustrated

[1]G.G. Stokes, On the theory of oscillatory waves. Camb Trans 8:441–473.

by a series of examples on gyroscopic motion, control problems, and a model of thermal explosion. These problems can be quite difficult, so much of the detailed calculation is given. In Chap. 8 the concepts of canard and black swan are introduced and illustrated by examples on the van der Pol oscillator, a fast phages–slow bacteria model, and some laser and chemical models. There is also a detailed discussion of two classical combustion models, including the calculation of the critical value of the parameter that separates explosive from non-explosive regimes. In Chap. 9 the proofs of certain theorems are given that have been signalled earlier in the book. These require a more mature reader.

The authors are grateful to Robert O'Malley who was there at the beginning and gave much valuable advice, as well as Grigory Barenblatt, Eric Benoit and Jean Mawhin for helpful discussions. This work is supported in part by the Russian Foundation for Basic Research (grants 12-08-00069, 13-01-97002, 14-01-97018, 14-08-91373), TUBITAK (grant 113E595), Division on the EMMCP of Russian Academy of Sciences, Program for basic research no. 14, project 1.12, and the Ministry of Education and Science of the Russian Federation in the framework of the implementation of Program of increasing the competitiveness of SSAU for 2013–2020 years.

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