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Differential Characters

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Preface

This text provides a systematic introduction to differential characters, as introduced by Cheeger and Simons. Differential characters form a model of what is nowadays called differential cohomology. In degree 2, integral cohomology of a space X classifies $\mathrm{U}(1)$-bundles over X via the first Chern class, while differential characters correspond to $\mathrm{U}(1)$-bundles with a connection. Similarly, in degree 3, integral cohomology classes classify gerbes over X while differential characters correspond to gerbes with additional geometric structure.

We construct the product which provides differential cohomology with a ring structure and we describe the fiber integration map. In both cases, we show uniqueness in the sense that these operations are determined by certain natural axioms. This shows in particular that the various very different descriptions in the literature are equivalent. We present natural and explicit geometric formulas for both the product and the fiber integration map.

The underlying space X may be more general than a finite-dimensional manifold. We allow for “smooth spaces” which contains loop spaces of manifolds, for instance. This is important for applications like the transgression map.

Up to now, there does not exist much literature on the relative version of differential characters. We investigate them in detail. In degree 2, a relative differential character corresponds to a $\mathrm{U}(1)$-bundle with connection and a section over a subspace. We derive long exact sequences which relate absolute and relative differential characters. Fiber integration for fibers with boundary is naturally considered in the relative framework. The module structure of relative differential cohomology over the ring of absolute differential characters is derived.

We discuss various applications including chain field theories and higher dimensional holonomies which occur as actions in string theory.

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