

Editors-in-Chief:

J.-M. Morel, Cachan

B. Teissier, Paris

Advisory Board:

Camillo De Lellis (Zürich)

Mario Di Bernardo (Bristol)

Alessio Figalli (Austin)

Davar Khoshnevisan (Salt Lake City)

Ioannis Kontoyiannis (Athens)

Gabor Lugosi (Barcelona)

Mark Podolskii (Aarhus)

Sylvia Serfaty (Paris and NY)

Catharina Stroppel (Bonn)

Anna Wienhard (Heidelberg)

For further volumes:

<http://www.springer.com/series/304>



Fondazione C.I.M.E., Firenze

C.I.M.E. stands for *Centro Internazionale Matematico Estivo*, that is, International Mathematical Summer Centre. Conceived in the early fifties, it was born in 1954 in Florence, Italy, and welcomed by the world mathematical community: it continues successfully, year for year, to this day.

Many mathematicians from all over the world have been involved in a way or another in C.I.M.E.'s activities over the years. The main purpose and mode of functioning of the Centre may be summarised as follows: every year, during the summer, sessions on different themes from pure and applied mathematics are offered by application to mathematicians from all countries. A Session is generally based on three or four main courses given by specialists of international renown, plus a certain number of seminars, and is held in an attractive rural location in Italy.

The aim of a C.I.M.E. session is to bring to the attention of younger researchers the origins, development, and perspectives of some very active branch of mathematical research. The topics of the courses are generally of international resonance. The full immersion atmosphere of the courses and the daily exchange among participants are thus an initiation to international collaboration in mathematical research.

C.I.M.E. Director

Pietro ZECCA

Dipartimento di Energetica "S. Stecco"

Università di Firenze

Via S. Marta, 3

50139 Florence

Italy

e-mail: zecca@unifi.it

C.I.M.E. Secretary

Elvira MASCOLO

Dipartimento di Matematica "U. Dini"

Università di Firenze

viale G.B. Morgagni 67/A

50134 Florence

Italy

e-mail: mascolo@math.unifi.it

For more information see CIME's homepage: <http://www.cime.unifi.it>

Aldo Conca • Sandra Di Rocco • Jan Draisma •
June Huh • Bernd Sturmfels • Filippo Viviani

Combinatorial Algebraic Geometry

Levico Terme, Italy 2013

Editors: Sandra Di Rocco, Bernd Sturmfels

In collaboration with



Springer



FONDAZIONE
CIME
ROBERTO CONTI

CENTRO INTERNAZIONALE MATEMATICO ESTIVO
INTERNATIONAL MATHEMATICAL SUMMER CENTER

Aldo Conca
Dipartimento di Matematica
Università di Genova
Genova, Italy

Sandra Di Rocco
Department of Mathematics
KTH Royal Institute of Technology
Stockholm, Sweden

Jan Draisma
Department of Mathematics
and Computer Science
TU Eindhoven
Eindhoven, The Netherlands

June Huh
Department of Mathematics
University of Michigan at Ann Arbor
Ann Arbor, MI, USA

Bernd Sturmfels
Department of Mathematics
University of California, Berkeley
Berkeley, CA, USA

Filippo Viviani
Dipartimento di Matematica
Università Roma Tre
Roma, Italy

ISBN 978-3-319-04869-7 ISBN 978-3-319-04870-3 (eBook)
DOI 10.1007/978-3-319-04870-3
Springer Cham Heidelberg New York Dordrecht London

Lecture Notes in Mathematics ISSN print edition: 0075-8434
ISSN electronic edition: 1617-9692

Library of Congress Control Number: 2014939301

Mathematics Subject Classification (2010): 11H55, 13D02, 13P25, 14H10, 14M25, 16S37, 52B20, 62F10

© Springer International Publishing Switzerland 2014

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

Combinatorics and Algebraic Geometry have enjoyed a fruitful interplay since the nineteenth century. Classical interactions include invariant theory, theta functions, and enumerative geometry. The aim of this volume is to introduce recent development in combinatorial algebraic geometry.

The five chapters of this book are based on the lectures delivered at the CIME-CIRM summer-school, Levico Terme, June 10–15, 2013.

We here regard algebraic geometry with a view towards applications, such as tensor calculus and algebraic statistics. A common theme is the study of algebraic varieties endowed with a rich combinatorial structure. Relevant techniques are polyhedral geometry, free resolutions, multilinear algebra, projective duality, and compactifications.

Aldo Conca offers an introduction to *Koszul Algebras and Their Syzygies*. Koszul algebras are fundamental in commutative algebra, and they have numerous applications in algebraic geometry. One application presented here is the study of Castelnuovo–Mumford regularity of projective varieties. Other results presented in this chapter concern syzygies of Koszul algebras, the Koszul property of Veronese algebras, and algebras in the theory of hyperspace arrangements.

Systems of polynomial equations in infinitely many variables arise naturally in applied algebraic geometry. Typically, these infinite-dimensional systems have a lot of symmetry, and, in favorable circumstances, one encounters *Noetherianity up to Symmetry*. Jan Draisma offers a glimpse on recent developments in this field. His chapter focuses on examples from algebraic statistics and on the combinatorics of well-quasi-ordered sets.

Maximum Likelihood Geometry studies the critical points of monomial functions over a variety inside the probability simplex. The number of complex critical points, known as its maximum likelihood degree, is a topological invariant. June Huh joined Bernd Sturmfels in writing a chapter, which introduces this theory and its statistical motivations. Many favorites from combinatorial algebraic geometry are featured: toric varieties, matroids, A-discriminants, Grassmannians, and determinantal varieties.

Sandra Di Rocco lectured on *Linear Toric Fibrations*, that is, toric varieties which are birational to projective toric bundles. On the combinatorial side, these correspond to Cayley polytopes, a class of highly structured lattice polytopes that has received much attention in the recent literature. This chapter presents geometrical phenomena, in algebraic geometry and neighboring fields, which are characterized by a Cayley structure.

Filippo Viviani takes the reader on *A Tour of Hermitian Symmetric Manifolds*. These are Hermitian manifolds which are homogeneous and such that every point has a symmetry preserving the Hermitian structure. Examples of such manifolds serve as moduli spaces in algebraic and analytic geometry. This chapter offers an introduction to several different perspectives from which Hermitian symmetric manifolds can be studied.

We thank the CIME foundation and the CIRM center for hosting the school and for their generous support. All of us had a wonderful time at Levico Terme. The beautiful scenery of Trentino made the mathematical interactions and the stimulating lectures even more enjoyable.

Stockholm, Sweden
Berkeley, CA
October 2013

Sandra Di Rocco
Bernd Sturmfels

CIME activity is carried out with the collaboration and financial support of:

- INdAM (Istituto Nazionale di Alta Matematica)
- MIUR (Ministero dell'Istruzione, dell'Università e della Ricerca)
- Ente Cassa di Risparmio di Firenze

Contents

Koszul Algebras and Their Syzygies	1
Aldo Conca	
Noetherianity up to Symmetry	33
Jan Draisma	
Likelihood Geometry	63
June Huh and Bernd Sturmfels	
Linear Toric Fibrations.....	119
Sandra Di Rocco	
A Tour on Hermitian Symmetric Manifolds	149
Filippo Viviani	