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Siegfried Bosch

Lectures on Formal and Rigid Geometry

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Preface

This volume grew out of lectures that I gave on several occasions. First versions of the manuscript were prepared as handouts for students and later, in 2005, became a preprint of the Collaborative Research Center *Geometrical Structures in Mathematics* at the University of Münster.

The present Lecture Notes Volume is a revised and slightly expanded version of the earlier preprint. Although I kept the lecture-style presentation, I added more motivation on basic ideas as well as some fundamental examples. To make the text virtually self-contained, the theory of completed tensor products was included in a separate appendix.

It is a pleasure for me to express my gratitude to students, colleagues and, particularly, to M. Strauch for their valuable comments and suggestions. Also I would like to thank the referees for their constructive remarks which, finally, made the text more complete and easier to digest.

Münster, Germany
February 2014

Siegfried Bosch

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