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Lectures on Mappings of Finite Distortion

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Preface

This material is based on the graduate level courses that the authors have given at the University of Michigan, University of Jyväskylä, and Charles University in Prague and on short courses by the authors at summer schools in Ischia and at the de Giorgi Center in Pisa. We thank the participants of these courses for their questions that have shaped the contents and for pointing out a number of mistakes in previous versions.

In order to make the topic accessible to a beginning graduate student, we have included a great number of details, especially in the first four chapters that can form a basis for a graduate course. Additionally, we have recorded all the necessary background material from real analysis and from the theory of Sobolev spaces that is not necessarily covered in undergraduate studies or in the basic graduate level real analysis courses. The later chapters partially cover very recent research, not included in the research monographs [4, 67] that we recommend for further reading.

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Contents

Notation

$\mathbf{N}$, $\mathbf{Z}$, $\mathbf{R}$	Positive integers, integers, real numbers
(a, b)	Open interval in $\mathbf{R}$ for $a, b \in \mathbf{R}$, $a < b$
$[a, b]$	Closed interval in $\mathbf{R}$ for $a, b \in \mathbf{R}$, $a < b$
$\mathbf{R}^n$	n-dimensional Euclidean space
$\|x\|$	The Euclidean norm of a vector $x \in \mathbf{R}^n$
$\\|x\\|$	In Chap. 4 we use this notation for maximum norm of $x \in \mathbf{R}^n$
$\langle u, v\rangle$	Usual inner product of vectors $u, v \in \mathbf{R}^n$, i.e. $\langle u, v\rangle = \sum_{i=1}^n u_i v_i$
$u \otimes v$	Tensor product of vectors $u, v \in \mathbf{R}^n$, i.e. $n \times n$ matrix $\{u_i v_j\}_{i,j=1}^n$
$\|E\|$	Operator norm of the matrix E, i.e. $\sup\{\|Ex\| : \|x\| \leq 1\}$
I	Identity matrix, i.e. 1 on the diagonal and 0 otherwise
$\operatorname{adj} E$	Adjoint matrix of the matrix E, i.e. $E \operatorname{adj} E = I \det E$
$B(c, r)$	Open ball centered at $c \in \mathbf{R}^n$ with radius $r > 0$, i.e. $\{x \in \mathbf{R}^n : \|x - c\| < r\}$
$Q(c, r)$	Open cube centered at $c \in \mathbf{R}^n$ with radius $r > 0$, i.e. $\{x \in \mathbf{R}^n : \\|x - c\\| < r\}$
$tB(c, r)$	Inflated ball for $t > 0$, i.e. $B(c, tr)$
$tQ(c, r)$	Inflated cube for $t > 0$, i.e. $Q(c, tr)$
$S^{n-1}(c, r)$	Sphere centered at $c \in \mathbf{R}^n$ with radius $r > 0$, i.e. $\{x \in \mathbf{R}^n : \|x - c\| = r\}$
ω_{n-1}	$(n-1)$-dimensional measure of $S^{n-1}(0, 1)$
$[x, y]$	Line segment connecting $x, y \in \mathbf{R}^n$
$\operatorname{dist}(x, A)$	Distance of a point $x \in \mathbf{R}^n$ to a set $A \subset \mathbf{R}^n$
$\operatorname{dist}(A, B)$	Distance of two sets $A,\ B \subset \mathbf{R}^n$
$\overline{A}$	Closure of a set $A \subset \mathbf{R}^n$
∂A	Boundary of a set $A \subset \mathbf{R}^n$
$\operatorname{diam} A$	Diameter of a set $A \subset \mathbf{R}^n$, $\operatorname{diam} A = \sup\{\|x - y\| : x, y \in A\}$
Ω	By Ω we always denote an open subset of $\mathbf{R}^n$
$A \subset\subset \Omega$	Set A is compactly contained in Ω, i.e. $\overline{A} \subset \Omega$ and $\overline{A}$ is compact
$\|A\|$ or $\mathscr{L}_n(A)$	n-dimensional Lebesgue measure of a measurable set A

χ_A	Characteristic function of a set A, i.e. 1 on A and 0 otherwise
$\#A$	Cardinality of the set A, i.e. the number of the elements in A
sgn	Sign function, i.e. $\operatorname{sgn} t = 1$ for $t > 0$, $\operatorname{sgn} t = -1$ for $t < 0$ and $\operatorname{sgn} 0 = 0$
$f^+(x)$	Nonnegative part of function f, i.e. $\max\{f(x), 0\}$
$\operatorname{spt} f$	Support of a function f, $\operatorname{spt} f = \overline{\{x : f(x) \neq 0\}}$
$L^p(\Omega)$	Lebesgue space—see Appendix for the definition
$\\|f\\|_p$, $\\|f\\|_{L^p}$	The L^p norm of function f
$L^p_{\mathrm{loc}}(\Omega)$	Local Lebesgue space—see Appendix for the definition
$L^n \log^\alpha L(\Omega)$	Zygmund space—see Definition 2.6
$WL^n \log^\alpha L(\Omega)$	Sobolev Zygmund space—see Definition 2.6
$W^{1,p}(\Omega)$	Sobolev space—see Appendix for the definition
$W^{1,p}_{\mathrm{loc}}(\Omega)$	Local Sobolev space—see Appendix for the definition
$W^{1,p}_0(\Omega)$	Sobolev space with zero boundary values—see Appendix for the definition
$BV(\Omega)$	The space of functions with bounded variation, see Definition 5.1
∇f	Classical derivative (gradient) of function $f : \Omega \to \mathbf{R}$ or mapping $f : \Omega \to \mathbf{R}^n$
Df	Weak derivative of function or mapping f—see Definition A.13
$J_f(x)$	Jacobian, i.e. the determinant of $Df(x)$ for $f = (f_1, \dots, f_n) : \Omega \to \mathbf{R}^n$. Sometimes we use $J(f_1, f_2, \dots, f_n)(x)$ to point out the components of f
K_f or K	Distortion of function f, see Definition 1.11
K_I	Inner distortion function, see Sect. 7.1
$\mathscr{J}_f$	Distributional Jacobian, see Sect. 2.2
$\deg(C, f, U)$	Topological degree of f on a set C with respect to U, see Sect. 3.2
$N(f, \Omega, y)$	Number of preimages of point y in Ω under f
C, C_C	Continuous functions, continuous functions with compact support
C^1 (C^2)	The class of functions with continuous first order (second order) derivatives
$C^\infty_C(\Omega)$	The class of compactly supported ($\operatorname{spt} f \subset\subset \Omega$) functions with derivatives of all orders
$C^\infty_0(\Omega)$	Functions whose extension by 0 to $\mathbf{R}^n \setminus \Omega$ belong to $C^\infty(\mathbf{R}^n)$
$f_A = \fint_A f$	Integral average of $f : A \to \mathbf{R}$ defined as $\frac{1}{\|A\|} \int_A f(x)\, dx$
$\operatorname{osc}_B f$	Oscillation of f on a set B, i.e. diameter of the image $\operatorname{osc}_B f = \operatorname{diam} f(B)$
Mf	Maximal operator of f, see Sect. 7.3
$\mathscr{H}^k$	The k-dimensional Hausdorff measure
$\mathscr{H}^k_\varepsilon$	Set functions in the definition of $\mathscr{H}^k$, i.e. $\mathscr{H}^k(A) = \lim_{\varepsilon \to 0+} \mathscr{H}^k_\varepsilon(A)$

$\int_{S^{n-1}(c,t)} f$	Integral of f with respect to the surface measure (constant multiple of $\mathscr{H}^{n-1}$)
C	We use the usual convention that C denotes a generic positive constant whose exact value may change at each occurrence
$a \approx b$	Means that $a \leq Cb$ and $b \leq Ca$