Lecture Notes in Mathematics 2101

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Random Walks on Disordered Media and their Scaling Limits

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Preface

The main theme of these lecture notes is to analyze heat conduction on disordered media such as fractals and percolation clusters by means of both probabilistic and analytic methods, and to study the scaling limits of Markov chains on the media.

The problem of random walk on a percolation cluster "the ant in the labyrinth" has received much attention both in the physics and in the mathematics literature. In 1986, H. Kesten showed an anomalous behavior of a random walk on a percolation cluster at critical probability for trees and for $\mathbb{Z}^2$. (To be precise, the critical percolation cluster is finite, so the random walk is considered on an incipient infinite cluster (IIC), namely a critical percolation cluster conditioned to be infinite.) Partly motivated by this work, analysis and diffusion processes on fractals have been developed since the late 1980s. As a result, various new methods have been produced to estimate heat kernels on disordered media, and these turn out to be useful to establish quenched estimates on random media. Recently, it has been proved that random walks on IICs are sub-diffusive on $\mathbb{Z}^d$ when d is high enough, on trees, and on the spread-out percolation for $d > 6$.

Throughout the lecture notes, I will survey the above-mentioned developments in a compact way. In the first part, I will summarize some classical and nonclassical estimates for heat kernels and discuss stability of the estimates under perturbations of operators and spaces. Here Nash inequalities and equivalent inequalities will play a central role. In the latter part, I will give various examples of disordered media and obtain heat kernel estimates for Markov chains on them. In some models, I will also discuss scaling limits of the Markov chains. Examples of disordered media include fractals, percolation clusters, random conductance models, and random graphs.

In summer 2013, I have made a revised final version of the notes. There has been significant progress in the areas for the 3 years after my St. Flour lectures. I have tried to include the information of recent developments as much as possible, but they are clearly far from complete. I also put several new ingredients concerning heat kernel estimates (Sects. 3.3 and 3.4).

For typos and errors, I will update a list of corrections at the following page.

http://www.kurims.kyoto-u.ac.jp/~kumagai/

Kyoto, Japan
September 2013

Takashi Kumagai

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The final corrections of the notes are made when I was visiting Max Planck Institute in Leipzig and Universität Leipzig. I would like to thank Max von Renesse and A. Sapozhnikov for their hospitality during my stay.

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