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Saint-Flour Probability Summer School



The Saint-Flour volumes are reflections of the courses given at the Saint-Flour Probability Summer School. Founded in 1971, this school is organised every year by the Laboratoire de Mathématiques (CNRS and Université Blaise Pascal, Clermont-Ferrand, France). It is intended for PhD students, teachers and researchers who are interested in probability theory, statistics, and in their applications.

The duration of each school is 13 days (it was 17 days up to 2005), and up to 70 participants can attend it. The aim is to provide, in three high-level courses, a comprehensive study of some fields in probability theory or Statistics. The lecturers are chosen by an international scientific board. The participants themselves also have the opportunity to give short lectures about their research work.

Participants are lodged and work in the same building, a former seminary built in the 18th century in the city of Saint-Flour, at an altitude of 900 m. The pleasant surroundings facilitate scientific discussion and exchange.

The Saint-Flour Probability Summer School is supported by:

- Université Blaise Pascal
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For more information, see back pages of the book and
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Coarse Geometry and Randomness

École d'Été de Probabilités
de Saint-Flour XLI – 2011



Springer

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ISBN 978-3-319-02575-9 ISBN 978-3-319-02576-6 (eBook)
DOI 10.1007/978-3-319-02576-6
Springer Cham Heidelberg New York Dordrecht London

Lecture Notes in Mathematics ISSN print edition: 0075-8434
ISSN electronic edition: 1617-9692

Library of Congress Control Number: 2013953863

Mathematics Subject Classification (2010): 82B43, 82B41, 05C81, 05C10, 05C80

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Printed on acid-free paper

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Preface

The first part of the notes reviews several coarse geometric concepts. We will then move on and look at the manifestation of the underlying geometry in the behavior of random processes, mostly percolation and random walk.

The study of the geometry of infinite vertex transitive graphs and Cayley graphs in particular is rather well developed. One goal of these notes is to point to some random metric spaces modeled by graphs that turn out to be somewhat exotic. That is, admitting a combination of properties not encountered in the vertex transitive world. These include percolation cluster on vertex transitive graphs, critical clusters, local and scaling limits of graphs, long range percolation, CCCP graphs obtained by contracting percolation clusters on graphs, and stationary random graphs including the uniform infinite planar triangulation (UIPT) and the stochastic hyperbolic planar quadrangulation.

Chapter 5 is due to Nicolas Curien, Chap. 12 was written by Ariel Yadin, and Chap. 13 is joint work with Gady Kozma.

I would like to deeply thank Omer Angel, Louigi Addario-Berry, Agelos Georgakopoulos, and Vladimir Shchur for comments, remarks, and corrections, and Nicolas Curien, Ron Rosenthal, and Johan Tykesson for *great help* with editing, collecting, and joining together the material presented.

Some of the proofs will only be sketched, or left as exercises to the reader. References to where proofs can be found in full detail are given throughout the text. Exercises and open problems can be found in most sections.

Excellent sources covering related material are Lyons with Peres [Lyo09], Pete [Pet09], Peres [Per99], and Woess [Woe05].

Thanks to N., Jean Picard and the St. Flour school organizers.

Contents

1	Introductory Graph and Metric Notions	1
2	On the Structure of Vertex Transitive Graphs	19
3	The Hyperbolic Plane and Hyperbolic Graphs	23
4	Percolation on Graphs	33
5	Local Limits of Graphs	41
6	Random Planar Geometry.....	53
7	Growth and Isoperimetric Profile of Planar Graphs	59
8	Critical Percolation on Non-Amenable Groups	63
9	Uniqueness of the Infinite Percolation Cluster	69
10	Percolation Perturbations	85
11	Percolation on Expanders	97
12	Harmonic Functions on Graphs	107
13	Nonamenable Liouville Graphs.....	121
	References.....	125