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Sadrilla Abdullaev

Magnetic Stochasticity in Magnetically Confined Fusion Plasmas

Chaos of Field Lines and Charged Particle Dynamics

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To my parents Sayfulla Abdullaev and Nuri Adilova

Preface

The magnetic confinement concept of thermonuclear plasma is one of the areas of controlled fusion research developing in the past 50 years in a quest for clean and unlimited source of energy. The concept is based on the fact that charged particles predominantly follow the magnetic field and therefore the magnetic field localized in a finite area would be able to contain the hot temperature plasma is a spatially bounded area. In magnetic confinement fusion devices, tokamaks and stellarators, the confinement of charged particles has been achieved by the magnetic fields whose field lines lie on the nested (magnetic) toroidal surfaces. However, from the beginning of these studies it was realized that the magnetic field created in real devices may deviate from the intended ideal toroidal configuration due to technical imperfections or plasma instabilities, which eventually may break the symmetry of magnetic field leading to the destruction of magnetic surfaces and plasma confinement. Therefore the problem of stability of the magnetic surfaces with respect to the small deviations (or perturbations) of the magnetic field from the ideal toroidal configuration had become one of the most important issues in the fusion research.

The important fact used in these studies was that a divergence-free magnetic field is equivalent to a Hamiltonian system with $1 + 1/2$ degrees of freedom. Thus the problem of stability of magnetic surfaces has been reduced to the classical problem of the stability of Hamiltonian dynamical systems which has been a subject of many studies in classical and celestial mechanics in the nineteenth and twentieth centuries. Already at that time H. Poincaré noted that the prediction in dynamical systems may become impossible and this problem is not related to the universal laws of motion, but with the specification of the initial conditions, *it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error [change] in the former will produce an enormous error [change] in the latter. Prediction becomes impossible* . . . [Poincaré (2009)]. In the modern terminology he discovered the chaotic motion in dynamical systems, now known as a *dynamical chaos*.

Until the 1960s the theory of dynamical systems has been primarily a research area of mathematicians. In the middle of the twentieth century researchers in fusion and accelerator physics revived the interest to this area of classical physics among the physical community whose interest until the 1960s has been mostly occupied by quantum physics. These studies had a crucial impact on the development of the theory of nonlinear dynamics and chaos in physics and beyond it. Particularly, the qualitative criteria of the onset of chaotic motion in physical systems proposed by Chirikov (1959) laid the foundation for the physical theory of chaos.

The phenomenon of chaos of magnetic field lines known as *magnetic stochasticity* (or *magnetic chaos*) plays an important role in the magnetic confinement of fusion plasmas. It concerns with the problems of stability and destruction of the magnetic field surfaces, chaotic behavior of magnetic field lines, and related transport of energy and particles.

In early studies the magnetic stochasticity in fusion devices is considered an undesirable effect which deteriorates the plasma confinement due to the enhanced radial transport of particles and energy along the chaotic field lines. However, at the end of the 1970s it was realized that the phenomenon of magnetic stochasticity can be used to control the transport of energy and particles. Particularly, the *ergodic divertor* concept has been proposed to divert particles and a heat releasing from the plasma to special plates in a controlled way by externally imposed magnetic perturbations which have been implemented in several tokamaks.

At present a magnetic stochasticity has been used that mitigates some undesirable phenomena developing in modern fusion devices. Among them the so-called edge localized modes, known as repetitive heat and particle loading to the divertor targets produced by plasma instabilities at the plasma edge in H-mode regimes. The suppression of these modes is of paramount importance for the International Thermonuclear Experimental Reactor (ITER) to protect wall materials from the damaging effect of huge heat and particles releasing during its operation. The suppression of runaway electrons generated in tokamaks during disruptions of plasma discharges by magnetic perturbations is another example of application of magnetic stochasticity. The runaway electrons with energies up to several tens of MeV may cause severe damage to walls.

These applications show the importance of stochastic magnetic fields to control plasmas in magnetically fusion devices. Therefore, a deeper knowledge of a magnetic stochasticity, its onset, and generic properties would be useful for researchers involved in magnetically fusion studies. There are many textbooks and monographs on chaotic dynamics in which the problems of magnetic stochasticity are presented. However, these presentations are too general and do not take the important features of a magnetic field in fusion devices. At the same time in many books on plasma physics the problem of magnetic stochasticity has been mentioned very briefly and treated only at the elementary level. This description of magnetic field stochasticity became insufficient to describe the present-day experimental observations of very fine patterns of plasma structures related to chaotic magnetic fields. These structures are manifestations of such phenomena as a splitting of separatrices, stable and unstable manifolds, introduced in the mathematical theory of dynamical systems.

The Aim of the Book

This book is intended to fill this gap and to give a systematic theoretical description of magnetic field stochasticity and related charged particle dynamics in magnetically confined fusion devices. The presentation is based on the Hamiltonian formulation of magnetic field lines and charged particles. It employs the classical mathematical tools as well as newly developed methods of Hamiltonian dynamics to study magnetic field lines and particle dynamics in toroidally confined plasmas.

The main intention of this book has been to present generic features of magnetic stochasticity in toroidal plasmas rather than its specific manifestations in a particular fusion device. At present they can be studied by the numerical codes for field line tracing. The knowledge of generic properties of a magnetic stochasticity is much useful to predict and, possible, to effectively control plasma behavior by applied magnetic perturbations. Particularly, we have made the main emphasis on revealing generic and universal features of magnetic perturbations generated by external coils in toroidal plasmas, the properties of chaotic magnetic field lines, and related transport of particles. The asymptotic and mapping methods are intensively employed to describe these generic features of magnetic fields and and their properties.

The choice of materials and the manner of presentation are somehow subjective. The Author tried to cover many issues of a magnetic stochasticity which are well understood. These problems were also studied in tokamaks which are considered the most advanced magnetically confined fusion device. However, some important issues of magnetic stochasticity related to the plasma response to applied magnetic perturbations, the screening of magnetic perturbations, nonlinear transport were outside the scope of the book. At present these problems are not well understood, and they are still under active research.

Structure of the Book

The book consists of 11 chapters and 7 appendices. Chapter 1 presents the Hamiltonian formulation of the equations of magnetic field lines based on the action principle of the classical mechanics. The different forms of the Hamiltonian equations for the magnetic field lines are derived. The magnetic flux coordinates are introduced using the action-angle formalism of classical mechanics. We have considered a model of magnetic field which is used to illustrate the action–angle formalism to study magnetic field lines.

The different analytical models of equilibrium plasmas are presented in Chap. 2. Besides the analytical models for plasmas with circular cross-sections and elongated shapes with magnetic separatrices, we have also described wire models of plasmas. The generic behavior of a magnetic field near the magnetic separatrix and the X-point are also analyzed. Finally, a wire model of a magnetic field with the snowflake divertor configuration is analyzed.

Chapter 3 presents the Hamiltonian equations of field lines in the presence of magnetic perturbations. Particularly, the generic properties of the spectra of magnetic perturbations created by sets of helical coils and saddle coils are analyzed. The corresponding asymptotical formulas for the spectra of the magnetic perturbations are obtained.

The Hamiltonian equations describing a guiding–center motion of charged particles in a toroidal system are presented in Chap. 4. We give an alternative formulation of these equations which uses the cylindrical coordinate system instead of magnetic flux coordinates used in the traditional approach. The Hamiltonian approach is used in Chap. 5 to study the guiding center orbits in equilibrium plasmas. We analyze drift surfaces of passing and trapped particles, and estimate their widths with a number of illustrations. The formulation of Hamiltonian equations in action–angle variables will be given. The behavior of guiding-center orbits near hyperbolic fixed points will be analyzed.

The modern methods to study the dynamics of Hamiltonian systems are presented in Chap. 6. First, we describe the mapping method to integrate Hamiltonian equations based on the canonical transformation of variables. Particularly, the explicit forms of mappings for magnetic field lines are written down. Then we construct the full-turn transfer mapping, the Poincaré mapping. The Poincaré and the Melnikov integrals associated with the full-turn transfer mappings will be introduced and their properties analyzed.

The mathematical aspects of onset of dynamical chaos in Hamiltonian systems are given in Chap. 7. Particularly, we formulate the Kolmogorov's theorem on the conservation of conditionally periodic motion and discuss its implications to a magnetic field system and charged particle motion in a magnetic field. The main obstacles to the integrability of Hamiltonian systems, the creations of isolated periodic orbits, the splitting of separatrices are discussed in detail. They are demonstrated on simple examples. Finally, the onset of chaotic motion due to the creation of infinite number of unstable periodic points and the homoclinic structure of phase space is discussed.

The physical theory of chaos in Hamiltonian systems is presented in Chap. 8. We discuss a nonlinear resonance phenomenon, its features in a magnetic field system, the formation of chaotic layer. The Chirikov's criterion of the onset of global chaos is discussed. We analyze a chaotic layer, its rescaling invariance properties near the hyperbolic fixed points, and their implications on the related transport in plasmas. The features of the onset of chaos in a magnetic system with a reversed shear are also briefly discussed.

The main features of magnetic fields in tokamaks induced by resonant magnetic perturbations are discussed in Chap. 9. First, we study the ergodic divertor concept to control the transport of heat and particle at the plasma edge. Particularly, the implementation of this concept in the Tore Supra and the TEXTOR tokamaks are presented. Then we discuss the features of a magnetic field in the DIII-D tokamak in the presence of magnetic perturbations created by a set of saddle coils. The models of magnetic perturbations in tokamaks with ergodic divertors and set of saddle coils are proposed. We also discuss the generic fractal properties of chaotic field lines at the plasma edge. Finally, some experimental observations of plasma structures induced by resonant magnetic perturbations are briefly discussed.

Chapter 10 describes the transport of field lines and charged particles in a stochastic magnetic field induced by a resonant magnetic perturbations. First, we briefly recall the basics of the kinetic description of the chaotic transport in Hamiltonian systems, particularly, in magnetic system with chaotic field lines. The diffusion of particles in a stochastic magnetic field in collisionless and collisional regimes is studied using analytical and numerical tools. A simple empirical formula for the collisional radial diffusion coefficient of particles in a stochastic magnetic field is presented.

Particle transport in Hamiltonian systems perturbed by a weak turbulent wave field and the transport of runaway electrons caused by a small-scale turbulent magnetic field in tokamaks are studied in Chap. 11. Particularly, we study the fractal nature of radial diffusion of particles and the formation of transport barriers near the low-order rational drift surfaces.

The analytical calculations of magnetic fields generated by sets of helical coils in the dynamic ergodic divertor of the TEXTOR tokamak and in the ergodic divertor of the Tore Supra tokamak are presented in Appendix A. The corresponding calculations of a magnetic field created by a set of saddle coils are given in Appendix B. The method of calculations of the Poincaré integrals for the magnetic field model is presented in Appendix C. In Appendix D the advanced version of the mapping for Hamiltonian systems is derived. The calculations of the Jacobi matrix and its eigenvalues are given in Appendix E. The features of the perturbation Hamiltonian of guiding-center motion in the presence of magnetic perturbations created by a set of saddle coils are studied in Appendix F. The derivation of the full-turn transfer mapping in a many-dimensional Hamiltonian system is described in Appendix G.

Most of the chapters are supplied with bibliographic notes or comments, in which the important references on subjects discussed there are given.

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Jülich, Germany, May 2013 Sadrilla Abdullaev

Contents

Main Notations

(R, Z, φ)	Cylindrical coordinates
(r, θ, φ)	Quasitoroidal coordinates (Fig. 1.2)
$\mathbf{A}$	Vector potential of a magnetic field
(A_R, A_Z, A_φ)	Components of a vector potential
$\mathbf{B}$	Magnetic field vector
(B_R, B_Z, B_φ)	Components of a magnetic field
R_0	Major radius of torus center
B_0	Magnitude of the toroidal magnetic field at the torus center
I_p	Plasma current
ρ	Minor radius of magnetic surfaces
$\psi \equiv \psi_\varphi$	Toroidal magnetic flux
$\psi_t \equiv \psi_\vartheta$	Poloidal magnetic flux
ψ_N	Normalized poloidal flux (Eq. 2.36)
ϑ	Poloidal angle in which field lines are straight (Eq. 1.32)
φ	Toroidal angle
$q(\psi)$	Safety factor
(m, n)	Poloidal and toroidal mode numbers in (Eq. 3.13)
γ	Coefficient in the asymptotics of $q(\psi)$ in (Eq. 2.51)
$\in$	Perturbation parameter (Eq. 3.11 and Eq. 3.46)
$R_n(\psi) = K_n(\psi) + iL_n(\psi)$	Fourier the coefficients (Eq. 6.77) of the Poincaré integral (Eq. 6.76)
c	Speed of light in vacuum
m_0	Particle mass (used only in Chaps. 4 and 5)
e	Elementary charge
Z_q	Particle charge (in unit e)
$\omega_c = eB_0/m_0c$	Reference gyrofrequency
$E_{ref} = m_0\omega_c^2R_0^2$	Reference energy
ε_0	Normalized energy of a particle at rest (Eq. 4.33)

γ_t	Relativistic factor (Eq. 4.6)
λ_I	Ratio of the action of radial gyro–oscillations I_x to the kinetic energy T_k (Eq.5.3)
$\chi_\parallel, \chi_\perp$	Parallel and perpendicular diffusion coefficients of particles due to collisions
D_{FL}	Diffusion coefficient of field lines
λ_{mfp}	Mean free path