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Martin Burger • Andrea C.G. Mennucci  
Stanley Osher • Martin Rumpf

# Level Set and PDE Based Reconstruction Methods in Imaging

Cetraro, Italy 2008

Editors: Martin Burger  
Stanley Osher



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**CIME**  
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# Preface

The present volume collects the lecture notes of courses given at the CIME Summer School in Cetraro in September 2008. The school appeared to be a highly successful event with around 50 participants from all over the world, enjoying both the scientific content of the courses and the atmosphere and opportunities for discussions among participants and with the speakers.

Mathematical imaging and inverse problems are two of the fastest growing disciplines in applied and interdisciplinary mathematics, with a strong overlap since many image analysis and reconstruction problems are effectively formulated as inverse problems. A key issue in such problems is to preserve and analyze edges or shapes in the image. Those are taken care by the methods covered in the summer school, which had the particular aim of providing a unified picture to methods of image reconstruction and analysis, on the one hand, and techniques for shape reconstruction and analysis, on the other hand. The key steps connecting those are methods based on level set representations and geometric partial differential equations, which effectively operate on the level sets (respectively, discontinuity set) of an image as well as on a single shape. The subject of lectures were chosen to cover these aspects and provide an outlook to other relevant topics:

- *Computational methods for nonlinear PDE and LI techniques*: Stanley Osher (UCLA).
- *Total variation and related methods*: Martin Burger (Münster).
- *The use of level set methods in inverse problems*: Oliver Dorn (Manchester).
- *Variational methods in image matching and motion extraction: Medical and biological applications*: Martin Rumpf (Bonn).
- *Metrics of curves in shape optimization and analysis*: Andrea C.G. Mennucci (SNS Pisa).

In addition, few young participants already involved in research related to the course topics were given an opportunity to present their work in shorter talks.

The lecture series by Stanley Osher gave a general overview to modern computational methods for image reconstruction and analysis, including Bregman iterations,

sparsity-based methods with  $\ell^1$ -type functionals, and local and nonlocal total variation methods for image denoising.

Martin Burger further expanded on total variation techniques and discussed the analysis of total variation methods, further details on Bregman iterations and inverse scale space methods, and also applications in biomedical imaging.

The lectures by Oliver Dorn gave an overview of level set methods for the reconstruction of shapes in inverse problems. He discussed basics about level set representation of shapes and the construction of computational methods to solve appropriate variational formulations, including the necessary shape calculus. A major part of the lecture series was devoted to several applications modeled by inverse problems for partial differential equations.

Martin Rumpf moved from single images to pairs of images. The major tasks then consist of finding deformations of the image domains that make the images look as similar as possible. The corresponding objectives are either matching if interested in resulting images, and motion estimation if interested in the deformation. Extensions of the framework to shapes and their matching respectively averaging were presented as well.

Andrea C.G. Mennucci provided an overview of metrics on curves, linking this topic originating in differential geometry to very applied problems in image processing and shape optimization. He showed that by choosing different metrics methods for tasks such as object segmentation in images can be improved.

In the lecture notes most topics of the first two lectures were unified in a chapter providing a comprehensive introduction to the zoo of total variation and related methods. More specialized topics related to photon count data frequently arising in imaging were unified with the contents of participant talks by Christoph Brune and Alex Sawatzky in the second chapter. The third chapter covers the contents of Martin Rumpf's lecture series, the fourth chapter the ones of Andrea C.G. Mennucci's. We hope that readers find the material as interesting as we do and enjoy the breadth of covered topics as well as the depth of detail in the single chapters.

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Los Angeles, CA

Martin Burger  
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