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Local Times and Excursion Theory for Brownian Motion

A Tale of Wiener and Itô Measures

Springer

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Preface

This monograph takes up and completes the volume written by the second author and edited by the University of Caracas (Venezuela), following a course given there in July 1994.

The present monograph consists essentially and naturally of three parts:

Part I presents local times for continuous semimartingales, while Part II is devoted to Excursion Theory for Brownian paths, and Part III to some applications of this theory. Chapter 1 gathers some facts which will be helpful throughout the volume.

However, this monograph differs from the Caracas volume in an essential way: the “credo” of the Caracas volume was that, once one knows “something” about the Wiener measure **W**, one should be able to translate this “something” in terms of Itô’s characteristic measure, usually denoted by **n**, for Brownian excursions.

The union of the translations of these “something,” such as the law under $\mathbf{n}(d\varepsilon)$ of the lifetime $V(\varepsilon)$ and/or the height $M(\varepsilon)$ of the generic excursion ε, should provide a full understanding of $\mathbf{n}(d\varepsilon)$ which, in turn, should enrich our understanding of Wiener measure $\mathbf{W}(dw)$.

In fact, as the second author experienced it, when teaching year after year this “local times—excursion” course: the reality is somewhat more complicated, it is true that D. Williams’ path decomposition of the excursion straddling the time $T_a = \inf\{t : B_t = a\}$ translates easily into Itô’s measure being disintegrated at the maximum of the height of the generic excursion, but, on the other hand, there is a priori no direct way to show that the normalized standard excursion, straddling deterministic time t, say, is equal (in law) to a BES(3) bridge. In fact, this result will follow from the disintegration of Itô’s measure **n**, with respect to its lifetime V.

It is this difficulty which led us to try and present an as easy as possible approach to both measures **W** and **n**, the fine descriptions of which being extremely intricate.

This central point (“the core of the course”) being explained, we refer the reader to the remaining plan of this volume, which is self-explanatory. Most proofs are self-contained, and references for the missing points are clearly indicated. For ease of the reader, each chapter has its own set of references, while general references are gathered at the end of the book, together with an index of terms.

Important Note: We consider these references, either at the end of a chapter, or at the end of the book, to convey some essential information for the reader. They may, or may not be cited in the text, but the reader is expected to consider them as a fertile source of material.

Beside this text, there exists a related one, by B. Mallein and the second author, where the same general thread is followed, but in a quite different manner: they only quote—without proofs—the main theorems of the different chapters, while the main body of each chapter consists in a number of exercises which were given, over the years, in exams related to this course.

This second volume may be a useful companion to the present one, with which the reader may hone his/her skills.

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