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Bruno Cordani

The Kepler Problem

Group Theoretical Aspects, Regularization and Quantization, with Application to the Study of Perturbations

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A Federica,

Violetta e Diletta

Preface

Because of the correspondences existing among all levels of reality,
truths pertaining to a lower level can be considered as symbols
of truths at a higher level and can therefore be the "foundation"
or support leading by analogy to a knowledge of the latter.
This confers to every science a superior or "elevating" meaning,
far deeper than its own original one.

— R. GUÉNON, The Crisis of Modern World

Having been interested in the Kepler Problem for a long time, I have always found it astonishing that no book has been written yet that would address all aspects of the problem. Besides hundreds of articles, at least three books (to my knowledge) have indeed been published already on the subject, namely Englefield (1972), Stiefel & Scheifele (1971) and Guillemin & Sternberg (1990). Each of these three books deals only with one or another aspect of the problem, though. For example, Englefield (1972) treats only the quantum aspects, and that in a local way. Similarly, Stiefel & Scheifele (1971) only considers the linearization of the equations of motion with application to the perturbations of celestial mechanics. Finally, Guillemin & Sternberg (1990) is devoted to the group theoretical and geometrical structure.

My aim in writing this book has been precisely to compensate for this lack. Its "completeness" can, undoubtedly, be questioned. My hope is, however, to cover the main aspects of the Kepler Problem. The methods presented here also lead to a computer program, named KEPLER, which allows one to calculate and display graphically the effects of a perturbation.

More generally, this book is hoped to illustrate the use of differential geometric methods in solving concrete physical problems, with the Kepler Problem playing the role of a sort of "phylogenetic recapitulation" of the mechanics.

I would like to express my gratitude to Prof. S. Sternberg for his helpful criticisms, suggestions and encouragement. Many thanks also to the anonymous referee: among other things, he suggested the title. P. Horváthy has helped to revise parts of the book. I am indebted to S. Benenti and his coworkers C. Chanu and G. Rastelli for improving Chapter 8 and, in particular, for their permission to reproduce a delicate, not yet published, proof. I had interesting discussions with F. Fassò and A. Giacobbe on action-angle variables, degenerate systems and monodromy. R. Cushman has pointed out to me some errors and imprecisions. Many thanks also to P. Casati and M. Tarallo for their accurate reading of Chapters 9 and 11 respectively, as well as to L. Galgani and K. Payne for having corrected many errors of my poor English. I had helpful discussions with A. Alzati, E. Colombo, G. Gaeta, F. Magri, I. Mladenov, C. Morosi, P. Nicola, M. Palleschi, M. Pedroni, C. Reina and V. Zambelli. I have received considerable help from three of my students: C. Passoni for his work on monodromy, G. Merlini who has written the VisualC++ code of KEPLER, and S. Codegoni who has written the EULER program. I would like to thank B. Ruf for support and help in publishing this book, and, last but not least, my colleague and friend E. Barazzetti, since also nonstrictly technical discussions may be illuminating.

But, above all, this book would never have been written without the support of my family. Only the love of my wife Federica and my daughters Violetta and Diletta has given me strength and serenity to work hard and to respond to various bitter disappointments with a smile and a shrug. To them, and in particular to my wife, is this book dedicated.

Milano, June 2002 B. CORDANI

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