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Probabilistic Group Theory, Combinatorics, and Computing

Lectures from the Fifth de Brún Workshop

Springer

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Preface

This book is inspired by the workshop *Groups, Combinatorics, Computing*, held at National University of Ireland, Galway from April 11 to 16, 2011—the Fifth "de Brún Workshop" run under the auspices of Science Foundation Ireland's Mathematics Initiative Programme. A principal theme of the workshop was interactions between group theory and combinatorics with algorithmic or computational aspects. Areas encompassed by this theme are currently the focus of intense research activity.

The core part of the workshop was formed by three lecture courses. These contained a wide and unique selection of material, for the first time providing an accessible introduction to frontier research in thematic areas. It became clear that the courses should be made available to a larger audience.

The book has three chapters, one per lecture course. Each chapter is self-contained; beginning with background material including historical roots, the reader is led to the latest results and open problems. Illustrative examples, some proofs and algorithms, and extensive bibliographies are given.

The first chapter, by Martin Liebeck, is an exposition of recent developments in probabilistic and asymptotic theory of finite groups, particularly finite simple groups. The first two sections are on random generation of finite groups and maximal subgroups. The next topic is representation varieties and character-theoretic methods. Finally, diameter and growth of Cayley graphs of simple groups are considered. The chapter traces progress on fundamental conjectures which have driven the development of this subject.

The second chapter is by Alice Niemeyer, Cheryl Praeger, and Ákos Seress. This chapter again has a strong probabilistic flavour. It discusses the role of estimation in the design and analysis of randomised algorithms for computing with finite groups, and approaches to estimating proportions of important element classes. Among the latter are geometric methods, the use of generating functions, and theory of Lie type groups. The chapter also surveys numerous results concerning estimation in permutation groups and finite classical groups. An application to the construction of involution centralisers, a key part of the constructive recognition of finite simple groups, is given. Connections with theoretical computer science are made.

In the final chapter, Leonard Soicher presents results from a different area at the interface of group theory and combinatorics. This chapter emphasises practical computation. Specifically, it considers how group theory may be used in the construction, classification, and analysis of combinatorial designs. Statistical optimality results for semi-Latin squares are reviewed. An account of the new theory of "uniform" semi-Latin squares and a construction which determines a semi-Latin square from a transitive permutation group are then given. The chapter describes use of the GAP package DESIGN. Along with an introduction to the package and samples of its operation, it is shown how package functions can be used to classify block designs and semi-Latin squares. In an extended example, new statistically efficient semi-Latin squares are determined.

We envisage that this book will be a resource for lecture or reading courses or for self-instruction. Indeed, each chapter is a ready-made graduate lecture course. All three chapters could serve as the foundation of an advanced graduate programme in algebra and computing.

The Fifth de Brún Workshop also featured research talks and short presentations. More details and pdf files of selected talks are available at

http://www.maths.nuigalway.ie/~detinko/DeBrun5/

We take this opportunity to record our gratitude to Charles Leedham-Green, who acted as scientific chair of the workshop. He gave the opening address, and his many other contributions helped to ensure the event's success.

In conclusion, we hope that the reader will find these lectures as interesting and valuable as workshop participants did.

Galway, Ireland — Alla Detinko and Dane Flannery
Auckland, New Zealand — Eamonn O'Brien
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