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# Mechanical Systems, Classical Models

## Volume I: Particle Mechanics

*by*

Petre P. Teodorescu
*Faculty of Mathematics,*
*University of Bucharest,*
*Romania*

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To
Oncuța,
Delia, Edu
and Ana,
my dear family

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# PREFACE

All phenomena in nature are characterized by motion; this is an essential property of matter, having infinitely many aspects. Motion can be mechanical, physical, chemical or biological, leading to various sciences of nature, mechanics being one of them. Mechanics deals with the objective laws of mechanical motion of bodies, the simplest form of motion.

In the study of a science of nature mathematics plays an important rôle. Mechanics is the first science of nature which was expressed in terms of mathematics by considering various mathematical models, associated to phenomena of the surrounding nature. Thus, its development was influenced by the use of a strong mathematical tool; on the other hand, we must observe that mechanics also influenced the introduction and the development of many mathematical notions.

In this respect, the guideline of the present book is precisely the mathematical model of mechanics. The classical models which we refer to are in fact models based on the Newtonian model of mechanics, on the five basic principles, i.e.: the inertia, the forces action, the action and reaction, the parallelogram and the initial conditions principle, respectively. Other models, e.g. the model of attraction forces between the particles of a discrete mechanical system are part of the considered Newtonian model. Kepler's laws brilliantly verify this model in case of velocities much smaller than the light velocity in vacuum. The non-classical models are relativistic and quantic.

Mechanics has as object of study mechanical systems; this notion is emphasized throughout the book, no matter the systems we are working with are discrete or continuous. We put into evidence the difference between these models, as well as the specificity of the corresponding studies; the generality of the proofs and of the corresponding computations yields a common form of the obtained mechanical results for both discrete and continuous systems. On the other hand, the discrete or continuous mechanical systems can be non-deformable (e.g., rigid solids) or deformable (deformable particle systems or deformable continuous media); for instance, the condition of equilibrium and motion, expressed by means of the "torsor", are necessary and sufficient in case of non-deformable and only necessary in case of deformable systems.

A special accent is put on the solving methodology as well as on the mathematical tool used; vectors, tensors and notions of the field theory. Continuous and discontinuous phenomena, various mechanical magnitudes are presented in a unitary form by means of the theory of distributions. Some appendices give the book an autonomy with respect to other works, special previous mathematical knowledge being not necessary.

Passing by non-significant details, one presents some applications connected to important phenomena of nature, and this also gives one the possibility to solve problems of interest from the technical, engineering point of view. In this form, the book becomes – we dare say – a unique outline of the literature in the field; the author wishes to present the most important aspects connected with the study of mechanical systems, mechanics being regarded as a science of nature, as well as its links to other sciences of nature. Implications in technical sciences are not neglected.

Starting from the particle (the simplest problem) and finishing with the study of dynamical systems (including bifurcation, catastrophes and chaos), the book covers a wide number of problems (classical or new ones), as one can see from its contents. It is divided in three volumes, i.e.: I. Particle mechanics. II. Mechanics of discrete and continuous systems. III. Analytical mechanics.

The book uses the known literature, as well as the original results of the author and his more than fifty years experience as a Professor of Mechanics at the University of Bucharest. It is devoted to a large circle of readers: mathematicians (especially those involved in applied mathematics), physicists (particularly those interested in mechanics and its connections), chemists, biologists, astronomers, engineers of various specialities (civil, mechanical engineers etc., who are scientific researchers or designers), students in various domains etc.

7 January 2006 P.P. Teodorescu