SpringerBriefs in Applied Sciences and Technology

For further volumes:
<http://www.springer.com/series/8884>

Emily S. C. Ching

Statistics and Scaling in Turbulent Rayleigh-Bénard Convection

Springer

Emily S. C. Ching
The Chinese University of Hong Kong
Hong Kong
Hong Kong SAR

ISSN 2191-530X ISSN 2191-5318 (electronic)
ISBN 978-981-4560-22-1 ISBN 978-981-4560-23-8 (eBook)
DOI 10.1007/978-981-4560-23-8
Springer Singapore Heidelberg New York Dordrecht London

Library of Congress Control Number: 2013944533

© The Author(s) 2014

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law. The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

When a fluid is heated, its density decreases due to expansion. Hotter fluid, therefore, has a tendency to rise and cooler fluid to fall. The onset of thermal convective motion occurs when the driving force due to the temperature difference is large enough to overcome the dampening effects of fluid viscosity and thermal diffusivity. Above the onset of convection, the velocity of the fluid increases with the temperature difference, and the convective fluid motion becomes turbulent when the velocity is sufficiently fast. Thermal convection is a major mechanism of heat transfer and is often used to provide cooling for equipment in engineering or industrial processes. Thermal convection is also ubiquitous in nature. It plays a role in the structure and dynamics of the Earth's atmosphere and in the earth's mantle, it is an important cause for motion of the tectonic plates. Most thermal convective flows in nature and technological applications are turbulent.

Experimental studies of thermal convection have often been carried out in the setting of Rayleigh-Bénard convection. The Rayleigh-Bénard convection system consists of a closed cell of fluid heated from below and cooled from above. Rayleigh-Bénard convection is a classical problem in fluid mechanics with many issues of interest. It is important for understanding stability of fluid flows and is a good paradigm for studying pattern formation near the onset of convection. In the turbulent regime, both velocity and temperature display complex fluctuations in time. The precise details of the fluctuations vary from flow to flow but the statistical characteristics of the fluctuations are reproducible. A key issue of interest is, therefore, to characterize and make sense of the statistics of the fluctuations. The statistics of the velocity and temperature differences, between measurements taken at two points separated by a distance l , can reveal the structure of turbulence. These structure functions often exhibit a power-law dependence on l , indicating scale invariance. Another issue of interest is to understand this scaling behavior. Amid the fluctuations, flow visualization reveals recurring flow structures that are persistent even after long-time averaging. Prominent examples include a large-scale mean circulating flow that spans the whole cell and thermal plumes, which are the flow structures generated by buoyancy near the top and bottom plates of the convective cell. The third issue of interest is to extract these flow structures from measurements and understand their dynamics and structure. Finally, it is of interest to understand how the heat transfer, which is greatly enhanced by the turbulent fluctuations, depends on the state of fluid flow.

In this monograph, we focus our discussion on the statistical properties and scaling behavior of the velocity and temperature fluctuations in three-dimensional turbulent Rayleigh-Bénard convection. In [Chap. 1](#), the problem of Rayleigh-Bénard convection, under the Oberbeck-Boussinesq approximation, is formulated. The derivation of two exact relations from the equations of motion is presented. In the turbulent regime, the velocity and the temperature fields display irregular fluctuations. In [Chap. 2](#), we introduce the basic statistical tools, including probability density functions (PDF) and conditional statistics, for studying fluctuations in general. As the equations of fluid motion are nonlinear, it is generally not feasible to directly calculate statistical quantities of interest from the equations of motion. This is known as the closure problem in turbulence. Because of this, exact implicit relations between different statistical quantities are useful. We derive two implicit formulae that relate the PDF of fluctuations to two conditional means, and discuss how these formulae have been applied to study the temperature fluctuations in turbulent Rayleigh-Bénard convection. In [Chap. 3](#), we introduce the important concept of energy cascade in turbulent flows and the different theories for the scaling behavior of the velocity and temperature fluctuations. We start with the scaling theory for non-buoyant turbulent flows and then discuss how the presence of buoyancy would affect and modify the scaling behavior. A crossover between the two types of scaling behavior is expected to occur at the length scale above which buoyancy is important. We present and discuss the experimental observations of the scaling behavior and examine the validity of these theoretical expectations in [Chap. 4](#). We end this monograph with a summary and outlook in [Chap. 5](#).

I would like to thank Leo Kadanoff and Albert Libchaber for introducing me to the subject, and acknowledge the fruitful collaborations and discussions with Roberto Benzi, Robert Kraichnan, Stephen B. Pope, Itamar Procaccia, Penger Tong, and Ke-Qing Xia.

Contents

1	The Rayleigh-Bénard Convection System	1
1.1	The Oberbeck-Boussinesq Equations	1
1.2	Boundary Conditions	3
1.3	Two Dimensionless Parameters: Rayleigh and Prandtl Numbers	4
1.4	Exact Relations	5
	References	8
2	Statistical Analysis of Turbulent Fluctuations	9
2.1	Basic Statistical Tools	9
2.1.1	Cumulative Distribution and Probability Density Function (PDF)	10
2.1.2	Moments and Characteristic Function	10
2.1.3	Some Common Examples of PDFs	12
2.1.4	Joint PDF and Conditional Probability	13
2.1.5	Random Functions	15
2.2	The Closure Problem of Turbulence	19
2.3	Exact Implicit PDF Formula	22
2.3.1	Stationary Fluctuations	22
2.3.2	Statistically Homogeneous Fluctuations	23
2.4	Some Exact Results for Conditional Means	24
2.4.1	Decaying Homogeneous Temperature Fluctuations	25
2.4.2	Stationary Homogeneous Temperature Fluctuations with a White-in-time Heat Source	26
2.5	Stationary Temperature Fluctuations in Rayleigh-Bénard Convection	28
2.5.1	The Central Region	28
2.5.2	Near the Bottom Plate	32
	References	34
3	Phenomenology and Scaling Theories	37
3.1	Richardson's Energy Cascade	37
3.2	The Kolmogorov 1941 Theory	38
3.3	The Four-Fifth Law	41
3.4	The Obukhov-Corrsin Theory for Passive Scalar	45

3.5	The Bolgiano–Obukhov Scaling	46
3.6	Crossover in Scaling	46
3.7	Refined Similarity Hypothesis.	47
3.8	Conditional Structure Functions.	49
	References	50
4	Observed Scaling Behavior.	51
4.1	Local Bolgiano Length.	51
4.2	Early Measurements in Time Domain	53
4.3	Central Region	54
4.4	Near the Top and Bottom Plates	55
	References	58
5	Summary and Outlook.	61
	References	64