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Takahiro Nemoto

Phenomenological Structure for the Large Deviation Principle in Time-Series Statistics

A Method to Control the Rare Events
in Non-equilibrium Systems

Doctoral Thesis accepted by
Kyoto University, Kyoto, Japan

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- T. Nemoto, Phys. Rev. E **85**, 061124 (2012).

Supervisor's Foreword

A large deviation function characterises the asymptotic behaviour of the probability of the average value for stochastic variables. The theory for large deviation functions was developed as a branch of probability theory about 50 years ago. Despite the progress in mathematics, many physicists are not so familiar with this large deviation theory. However, the situation is now changing. For a wide class of systems out of equilibrium, a novel symmetry property, which is now called the fluctuation theorem, was discovered. Such a simple and universal relation in physics was then clearly expressed by using the large deviation function of time-averaged entropy production rate. This example suggests that the study of fluctuations on the basis of the large deviation theory can discover new physical laws. Within this context, the Springer Thesis by Takahiro Nemoto provides us a thoughtful insight into the intersection between physics and the large deviation theory.

An important point of view is that the basic concept of large deviation theory was traced back to Einstein's fluctuation theory. On the one hand, large deviation functions are determined from fluctuation properties of thermodynamic extensive variables, and on the other hand, thermodynamic functions are obtained by the measurement of the heat capacity and the equation of state. Einstein's fluctuation theory begins with a hypothesis where these large deviation functions and thermodynamic functions are equivalent up to a multiplicative constant. This hypothesis could be proved on the principle of equal weight, from which many non-trivial predictions were confirmed experimentally, leading to the establishment of equilibrium statistical mechanics. Takahiro Nemoto seeks a similar framework for time-averaged quantities in systems out of equilibrium, by which large deviation functions can be obtained without measuring fluctuations. He calls such a framework "phenomenological structure for the large deviation principle" as an extension of Einstein's fluctuation theory.

Readers of this Springer Thesis can obtain the most advanced knowledge on the phenomenological structure for the large deviation principle with a background idea, a new theoretical method, novel formulas, and illustrative examples.

In addition to the fundamental aspects of the problem, this framework also involves a proposal of a new method for rare-event sampling, which could be another interesting topic. I am sure that graduate students in theoretical physics as well as researchers can enjoy reading this Springer Thesis.

Kyoto, Japan
September 2015

Prof. Shin-ichi Sasa

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