

# Statistical Learning with Math and R

Joe Suzuki

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100 Exercises for Building Logic

 Springer

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# Preface

I am currently with the Statistics Laboratory at Osaka University, Japan. I often meet with data scientists who are engaged in machine learning and statistical analyses for research collaborations and introducing my students to them. I recently found out that almost all of them think that (mathematical) logic rather than knowledge and experience is the most crucial ability for grasping the essence in their jobs. Our necessary knowledge is changing every day and can be obtained when needed. However, logic allows us to examine whether each item on the Internet is correct and follow any changes; without it, we might miss even chances.

In 2016, I started teaching statistical machine learning to the undergraduate students of the Mathematics Department. In the beginning, I was mainly teaching them what (statistical) machine learning (ML) is and how to use it. I explained the procedures of ML, such as logistic regression, support vector machines, k-means clustering, etc., by showing figures and providing intuitive explanations. At the same time, the students tried to understand ML by guessing the details. I also showed the students how to execute the ready-made functions in several R packages without showing the procedural details; at the same time, they understood how to use the R packages as black boxes.

However, as time went by, I felt that this manner of teaching should be changed. In other non-ML classes, I focus on making the students consider extending the ideas. I realized that they needed to understand the essence of the subject by mathematically considering problems and building programs. I am both a mathematician and an R/Python programmer and notice the importance of instilling logic inside each student. The basic idea is that the students see that both theory and practice meet and that using logic is necessary.

I was motivated to write this book because I could not find any other book that was inspired by the idea of “instilling logic” in the field of ML.

The closest comparison is “Introduction to Statistical Learning: with Application in R” (ISLR) by Gareth James, Daniela Witten, Trevor Hastie, and Robert Tibshirani (Springer), which is the most popular book in this field. I like this book and have used it for the aforementioned class. In particular, the presentation in the book is splendid (abundant figures and intuitive explanations). I followed this

style when writing this book. However, ISLR is intended for a beginner audience. Compared to ISLR, this book (SLMR) focuses more on mathematics and R programming, although the contents are similar: linear regression, classification, information criteria, regularizations, decision trees, support vector machine, and unsupervised learning.

Another similar book is “The Elements of Statistical Learning” (ESL) by Trevor Hastie, Robert Tibshirani, and Jerome Friedman (Springer), which provides the most reliable knowledge on statistical learning. I often use it when preparing for my classes. However, the volume of information in ESL is large, and it takes at least 500–1000 h to read it through, although I do recommend reading the book. My book, SLMR, on the other hand, takes at most 100 h, depending on the reader’s baseline ability, and it does not assume the reader has any knowledge of ML. After reading SLMR, it takes at most 300–500 h to read through ESL because the reader will have enough logic to easily understand ESL. ESL contains many equations and procedures but no programming codes.

In this sense, SLMR focuses on both mathematics and R programming more than ISLR and ESL. I sincerely wish that the reader of SLMR will develop both logic and statistical learning knowledge.

## What Makes SLMR Unique?

I have summarized the features of this book as follows.

1. Developing logic

To grasp the essence of the subject, we mathematically formulate and solve each ML problem and build those programs. The SLMR instills “logic” in the minds of the readers. The reader will acquire both the knowledge and ideas of ML so that even if new technology emerges, they will be able to follow the changes smoothly. After solving the 100 problems, most of the students would say “I learned a lot.”

2. Not just a story

If programming codes are available, you can immediately take action. It is unfortunate when an ML book does not offer the source codes. Even if a package is available, if we cannot see the inner workings of the programs, all we can do is input data into those programs. In SLMR, the program codes are available for most of the procedures. In cases where the reader does not understand the math, the codes will help them understand what it means.

3. Not just how to book: an academic book written by a university professor

This book explains how to use the package and provides examples of executions for those who are not familiar with them. Still, because only the inputs and outputs are visible, we can only see the procedure as a black box. In this sense, the reader will have limited satisfaction because they will not be able to

obtain the essence of the subject. SLMR intends to show the reader the heart of ML and is more of a full-fledged academic book.

4. Solve 100 exercises: problems are improved with feedback from university students

The exercises in this book have been used in university lectures and have been refined based on feedback from students. The best 100 problems were selected. Each chapter (except the exercises) explains the solutions, and you can solve all of the exercises by reading the book.

5. Self-contained

All of us have been discouraged by phrases such as “for the details, please refer to the literature XX.” Unless you are an enthusiastic reader or researcher, nobody will seek out those references. In this book, we have presented the material in such a way that consulting external references is not required. Additionally, the proofs are simple derivations, and the complicated proofs are given in the appendices at the end of each chapter. SLMR completes all discussions, including the appendices.

6. Readers’ pages: questions, discussion, and program files

The reader can ask any question on the book Facebook page (The URL will be set here). Additionally, all of the programs and data can be downloaded from <http://bitbucket.org/prof-joe> (thus, you do not have to copy the programs from the book).

7. Linear algebra

One of the bottlenecks in learning ML and statistics is linear algebra. Except for books for researchers, few books assume the reader has knowledge of linear algebra, and most books cannot go into the details of this subject. Therefore, SLMR contains a summary of linear algebra. This summary is only 14 pages and is not just an example, but it provides all the proofs. If you already know linear algebra, then you can skip it. However, if you are not confident in the subject, you can read in only one day.

## How to Use This Book

Each chapter consists of problems, their explanation (body), and an appendix (proof, program). You can start reading the body and solve the problem. Alternatively, you might want to solve the 100 exercises first and consult the body if necessary. Please read through the entire book until the end.

When used in a lecture, I recommend that the teacher organizes the class into 12-, 90-min lectures (or a 1000-min course) as follows: three lectures for Chaps. 1, 2 lectures for Chaps. 6, and 1 lecture for each of the other chapters. You may ask the students to complete the 100 exercises. If you read the text carefully, you will be

able to answer any of their questions. I think that the entire book can be fully read in about 12 lectures total.

Osaka, Japan  
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Joe Suzuki

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