Part IV
Advanced Problems

In Part IV, we introduce four topics of the computational origami. They are on the front-line of this research area. Comparing with Part II and Part III, they are somewhat limited or developing, rather meaningful to provide challenges that are expected to develop in the future. It is at the forefront of "just recently known", so the difficulty is unexpectedly disjointed. Some topics can be research themes at graduate schools; on the other hand, there are some themes that high school students can be likely to solve if good ideas emerge. It is a challenge to readers, in other words, I want you to have fun, troubles, or consider depending on the level of you.

The first two topics are related to the nets of polyhedra.

The first one is dealing with polyhedra such as triangular pyramids and quadrangular pyramids, that is, general pyramids. The research of this problem was initiated by myself when I was asked about the folding problem of triangular pyramids and quadrangular pyramids by one of my puzzle friends, Teruo Nishiyama, who is a former mathematics teacher. The nets of a triangular pyramid are relatively easy, but the problem becomes very difficult for a quadrangular pyramid or pyramids with more vertices. It is a simple and natural problem, and it seems that there can be existent some results; however, as far as I investigated, it does not seem to be well studied so far. In the chapter, we give the features of the nets of these general pyramids, consider the "bumpy" case where the bottom surface is not flat, and give some algorithmic solutions.

In the second topic, we investigate special nets which are unfolded by a method called "zipper unfolding". It may be imagined from the name, but in short, a zipper unfolded net is a net obtained by cutting along a line that you can draw in one stroke. That is, once you insert scissors in the polyhedron, you cut it along a line without branching off from it, and unfold it as it is. Since it can be realized by attaching only one zipper, it is called by such a name. In the chapter, let us give concrete examples of polyhedra that cannot be zipper unfolded, and introduce interesting open problems. This area may seem easy, but there are many problems that have not yet been solved.

The other two topics are related to computational complexity of the folding algorithm.

The third topic is a generalization of the crease width minimization problem of stamp folding of Sect. 6.2. In this generalization, the creases are set to general intervals instead of equal intervals. Originally, "crease width" is defined by the number

of sheets of paper layers at a crease; however, in general intervals, creases do not necessarily overlap with the same place anymore. Thus, it is necessary to generalize from the definition itself, and the property of the problem has changed a lot.

The last topic is an undecidable folding problem. This is a study of computability of a computation model, and it is a rather theoretical result that is possible to create an undecidable problem on origami when we introduce some computation model. It is recommended for readers who like arguments close to logic such as diagonalization and incompleteness theorem or pure mathematics. You may feel the depth of the notion of origami.