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Mathematical Modeling of Emission in Small-Size Cathode

Springer

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Preface

The study described in this book began several years ago when Prof. M. V. Karasev acquainted the authors with two MIET (National Research University "Moscow Institute of Electronic Technology") researchers, N. A. Dyuzhev and M. A. Makhiboroda who reported at the seminar that the mathematical modeling of electron emission from small-size cathodes has not yet been investigated full enough. The main problem in this study is the modeling of heat transfer with possible cathode melting.

The global (this term is explained in the book below) description of the heat transfer with melting taken into account can be obtained using the phase-field system, i.e., in the framework of the contemporary generalization of Stefan-type problems.

The phase-field model has already been studied during approximately 15 years in the mathematical literature but, as it turned out, the specialists in the field of electronics knew nearly nothing about it. The result of this model adoption is discussed in the book proposed to the reader.

It should be noted that the electron tunneling process was studied (mainly, from the physical viewpoint) in detail for semiconductors and metals in G. Fursey's book.[1] But the numerical modeling described in that book gave a very high temperature of the cathode but without taking the possibility of melting into account.

The proposed monograph mainly deals with mathematical modeling, namely it describes the mathematical model of heat transfer in a silicon cathode of small (nano) dimensions with the possibility of partial melting taken into account. This is a mathematically very difficult problem, and the properties of the solution of the phase-field system underlying our study have not yet been described completely. We present only well-known analytic results, but the basic idea in this book is the following one.

Instead of a pure mathematical approach, we apply an approach based on the understanding of the solution structure and computer simulation. The understanding of the solution structure is based on the construction of asymptotic solutions and an

[1]G. Fursey "Field Emission in Vacuum Microelectronics", Springer (2005).

analysis of their properties. The further comparison of the results of numerical experiments with the asymptotic solutions allows us to determine the process behavior and its characteristics.

Further, note that not all details of physical processes in the cathode (especially in the case of appearance of the liquid phase) have been described in the literature. Therefore, in numerical experiments, we made several additional assumptions. Moreover, in this book, we briefly consider some information given in the physical literature which (from our viewpoint) explains the properties of the mathematical model.

All physical parameters contained in our model can be found in the physical literature, i.e., in different reference books. The only exception is the expression describing the Nottingham effect. All physical constants in this expression are known, but the formula itself is obtained on the basis of tunneling through the potential barrier from half-plane to space. Thus, the actual geometry of the cathode is not taken into account in this formula. Usually, in problems of such a type, the cathode geometry is taken into account by using the so-called form-factor. We follow this way choosing a multiplier in the expression for the heat flow from the emitter surface. This multiplier is the only "adjusting" parameter in our problem and can easily be determined experimentally by comparing the theoretical value of the emission flow density (from the plane surface of the emitter) with the actual emission current observed in experiments.

The book is organized as follows. In Chap. 1, we describe the history of discovering the electron emission phenomenon and its types. We also present the mathematical statement of the problem of the field emission from semiconductor cathodes of small dimensions.

In Chap. 2, we briefly discuss some information from the solid-state physics, in particular, formulas for specific conductivity, thermoelectric height, and the Thompson coefficient for semiconductors. We also mathematically describe the tunneling process through the potential barrier on the cathode–vacuum interface, write a formula for the field emission current density in the case of metals, and describe its specific features in the case of field emission from semiconductor cathodes. At the end of the second chapter, we describe the Fowler–Nordheim theory and the Nottingham effect in the case of field emission from metals.

In Chap. 3, we discuss the phase-field model and the basic properties of its solutions. We also present formulas for asymptotic solutions of the phase-field system in the simplest case and in some special cases.

In Chap. 4, we present formulas for the numerical solution of the system of phase-field equations and an algorithm for solving the problem numerically (including its realization by contemporary multiprocessor systems and hybrid systems based on graphic accelerators) and discuss the results of numerical experiments.

Preface to the English Edition

In addition to the Preface to the Russian edition, we would like to make some remarks. The book deals with the physical problem—electron emission from a small-size cathode.

We initially intended to solve the problem of phase transition concerning the field emission of electrons from a small-size cathode. Obviously, the small size of the cathode and a relatively strong electric current density through it could result in melting of the cathode core. Thus, it was necessary to solve the Stefan problem taking into account the large curvature of the boundary between phases (the latter follows from the small size of the cathode).

If one begins to think about numerical calculations of the Stefan problem solution, then he immediately comes to the idea of adaptive mesh or something like this. A significant difference between geometric sizes (extremely small near the top of the cathode and relatively large near the bottom) results in additional troubles. Thus, to obtain the global solution of the phase-field system approximating the solution of the original problem seemed to be the most simple way to solve the problem.

We would like to stress that for the research work described in the book, we have to use relatively contemporary mathematical tools, especially the last results of the theory of generalized functions.

The most important point here is the construction of the definition of generalized solution to the phase-field system; see Sects. 3.4 and 4.6. As was discovered, the initial stage of melting required a special construction of liquid-phase nucleus for its description, where we actively used the concepts of mushy region and interaction of nonlinear waves; see Sect. 3.5.

And finally, we propose to use visualizations simultaneously with pure mathematical considerations to investigate the mathematical model; see Sect. 3.5.

The mathematical background is not the main part of this book, but, by our opinion, it would be better to learn about it before one starts his own investigations in the field described here.

For this edition, we checked and corrected many misprints and added some comments through out the text.

Moscow, Russia
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Roman Gaydukov
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