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Editor-in-Chief

Baoding Liu

Department of Mathematical Sciences

Tsinghua University

Beijing 100084, China

<http://orsc.edu.cn/liu>

Email: liu@tsinghua.edu.cn

Executive Editor-in-Chief

Kai Yao

School of Economics and Management

University of Chinese Academy of Sciences

Beijing 100190, China

<http://orsc.edu.cn/yao>

Email: yaokai@ucas.ac.cn

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Kai Yao

Uncertain Renewal Processes

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Kai Yao
School of Economics and Management
University of Chinese Academy of Sciences
Beijing, China

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*To My Wife
Meixia Wang
and
My Daughter
Anwen Yao*

Preface

The renewal process is used to count the renewals that occur in an indeterminate system. As the generalizations of renewal process, the renewal reward process models the cumulative rewards associated with the renewals up to some given time, and the alternating renewal process models the total on-time of an indeterminate system, which is on and off alternately.

In indeterminate systems, the interarrival times and the rewards could be random variables, uncertain variables, or uncertain random variables depending on that their values are assigned according to the frequency, the belief degree or both, thus resulting in the stochastic renewal systems, the uncertain renewal systems, and the uncertain random renewal systems.

This book aims at providing a comprehensive presentation of various types of indeterminate renewal processes. As the main contents, Chaps. 2, 4, and 6 introduce the renewal processes, the renewal reward processes, and the alternating renewal processes in the stochastic systems, in the uncertain systems and in the uncertain random systems, respectively. As the preliminaries to each case, Chaps. 1, 3, and 5 introduce some basic concepts and theorems in the probability theory, in the uncertainty theory and in the chance theory, respectively.

This book is suitable for researchers, engineers, and students in the field of mathematics, information science, operations research, industrial engineering, computer science, artificial intelligence, automation, economics, and management science. If you would like to read more papers related to uncertain renewal processes and uncertain random renewal processes as well as their applications, please visit the website at <http://orsc.edu.cn/online>.

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Beijing, China
May 2019

Kai Yao
<http://orsc.edu.cn/yao>

About This Book

Uncertain Renewal Processes

In the renewal theory, there are various types of renewal processes including the renewal process, the delayed renewal process, the renewal reward process, and the alternating renewal process. These renewal processes describe the renewals of an indeterminate system in different aspects.

In an indeterminate system, the interarrival times and the rewards could be random variables, uncertain variables, or uncertain random variables depending on their values assigned according to the frequency, the belief degree or both, thus resulting in the stochastic renewal processes, the uncertain renewal processes, and the uncertain random renewal processes.

This book provides a comprehensive presentation of the various types of indeterminate renewal processes. The probability theory, the uncertainty theory, and the chance theory are also introduced as the preliminaries to the stochastic renewal processes, the uncertain renewal processes, and the uncertain random renewal processes, respectively. This book shows applications of renewal processes in maintenance models and the insurance models.

Kai Yao

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Frequently Used Symbols

\Pr	Probability measure
$(\Omega, \mathcal{A}, \Pr)$	Probability space
ξ, η, τ	Random and/or uncertain variables
Φ, Ψ, Υ	Probability/uncertainty/chance distributions
ϕ, ψ	Probability density functions
E	Expected value
V	Variance
X_t, Y_t, Z_t	Stochastic and/or uncertain processes
N_t	Renewal process
R_t	Renewal reward process
A_t	Alternating renewal process
\mathcal{M}	Uncertain measure
$(\Gamma, \mathcal{L}, \mathcal{M})$	Uncertainty space
$\Phi^{-1}, \Psi^{-1}, \Upsilon^{-1}$	Inverse uncertainty distributions
Ch	Chance measure
\mathfrak{R}	The set of real numbers
\vee	Maximum operator
\wedge	Minimum operator