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Varsha Daftardar-Gejji
Editor

Fractional Calculus and Fractional Differential Equations

 Birkhäuser

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Foreword

It gives me immense pleasure to write the foreword for the volume edited by Prof. Varsha Daftardar-Gejji with contributions from eminent researchers in the fields of fractional calculus (FC) and fractional differential equations (FDEs). These are the most important and prominent areas for research which have emerged as interdisciplinary branch of mathematical, physical, biological sciences and engineering. This book provides a systematic, logical development of modern topics through the articles by eminent scientists and active researchers working in this area all over the globe.

Fractional calculus has a history of more than 300 years, while modelling of various phenomena in terms of fractional differential equations has gained momentum since the last two decades or so. There is an upsurge of research articles in the areas of FC and FDEs. This book is appealing and unique in this context as it encompasses numerical analysis of fractional differential equations, dynamics and stability analysis of fractional differential equations involving delay, variable-order fractional operators along with chapters on engineering applications. Moreover, the fractional analogues of classical Poisson processes, analysis of fractional differential equations using inequalities and comparison theorems are dealt with in a concise manner in this book.

Bologna, Italy

Francesco Mainardi
University of Bologna

Preface

Fractional calculus (FC) and fractional differential equations (FDEs) have emerged as the most important and prominent areas of interdisciplinary interest in recent years. FC has a history of more than 300 years, yet its applicability in different domains has been realised only recently. In the last three decades, the subject witnessed exponential growth and a number of researchers around the globe are actively working on this topic. The Department of Mathematics at Savitribai Phule Pune University (SPPU) organised a national workshop on fractional calculus in 2012, which was the first workshop in India that exclusively focussed on fractional calculus. This workshop attracted researchers of pure and applied mathematics, statisticians, physicists and engineers from all over India, working in fractional calculus and related areas. Deliberations in that workshop have been appeared earlier as a book titled *Fractional Calculus: Theory and Applications* which was very well received.

As a continuation of this, in 2017, we organised a national conference on fractional differential equations bringing together researchers in FDEs for academic exchange of ideas through discussions. Many active scientists from all parts of the country participated in this conference. It covered a significant range of topics motivating us to take up this endeavour. The present book comprises excellent contributions by the resource persons in this conference besides invited contributions from experts abroad, who willingly contributed. This book gives a panoramic overview of the latest developments and is expected to help new researchers entering this vast field.

The book comprises eight chapters which cover numerical analysis of FDEs, fractional Poisson processes, variable-order fractional operators, fractional-order delay differential equations and related phenomena including chaos, impulsive FDEs, inequalities and comparison theorems in FDEs. Moreover, artificial neural network and FDEs are also discussed by a group of engineers. New transform methods such as Sumudu transform methods are presented, and their utility for solving fractional partial differential equations (PDEs) is discussed.

The book is written keeping young researchers in mind who are planning to embark upon the research problems in FC and FDEs and related topics. There are many aspects that are still open for pursuing further research. If this book motivates some readers to venture into these areas, the aim of the endeavour will be fulfilled.

I am very grateful to all the researchers who have made wonderful contributions to this volume. My sincere thanks to Springer India Pvt. Ltd. for publishing this beautiful book. I also take this opportunity to thank the authorities of SPPU and my colleagues at the Department of Mathematics. Last but not least, my sincere thanks to my parents, husband and children for their unfailing support throughout.

Pune, India

Varsha Daftardar-Gejji

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About the Editor

Varsha Daftardar-Gejji is Professor at the Department of Mathematics, Savitribai Phule Pune University, India. She completed her Ph.D. at Pune University, India. She has developed original methods for solving fractional differential equations that have become widely popular. Her noteworthy contributions include analysis of fractional differential equations and developing theories of fractional-ordered dynamical systems and related phenomena such as chaos. She is the editor of the book *Fractional Calculus: Theory and Applications* and has co-authored the book *Differential Equations* (Schaum's Outline Series). She has published more than 65 papers in reputed international journals in areas of fractional calculus, fractional differential equations and general relativity.