

Fourier Analysis—A Signal Processing Approach

D. Sundararajan

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 Springer

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Preface

Transform methods dominate the study of linear time-invariant systems in all the areas of science and engineering, such as circuit theory, signal/image processing, communications, controls, vibration analysis, remote sensing, biomedical systems, optics, acoustics. The heart of the transform methods is Fourier analysis. Several other often used transforms are generalizations or specific versions of Fourier analysis. It is unique in that it is much used in theoretical studies as well as in practice. The reason for the latter case is the availability of fast algorithms to approximate the Fourier spectrum adequately. For example, the existence and continuing growth of digital signal and image processing are due to the ability to implement the Fourier analysis quickly by digital systems. This book is written for engineering, computer science, and physics students, and engineers and scientists. Therefore, Fourier analysis is presented primarily using physical explanations with waveforms and/or examples, keeping the mathematical form to the extent it is necessary for its practical use. In engineering applications of Fourier analysis, its interpretation and use are relatively more important than rigorous proofs. Plenty of examples, figures, tables, programs, and physical explanations make it easy for the reader to get a good grounding in the basics of Fourier signal representation and its applications.

This book is intended to be a textbook for senior undergraduate- and graduate-level Fourier analysis courses in engineering and science departments and a supplementary textbook for a variety of application courses in science and engineering, such as circuit theory, communications, signal processing, controls, remote sensing, image processing, medical analysis, acoustics, optics, and vibration analysis. For engineering professionals, this book will be useful for self-study. In addition, this book will be a reference for anyone, student or professional, specializing in the practical applications of Fourier analysis. The prerequisite for reading this book is a good knowledge of calculus, linear algebra, signals and systems, and programming at the undergraduate level.

Programming is an important component in learning and practicing Fourier analysis. A set of MATLAB[®] programs are available at the Web site of the book. While the use of a software package is inevitable in most applications, it is better to

use the software in addition to self-developed programs. The effective use of a software package or to develop own programs requires a good grounding in the basic principles of the Fourier analysis. Answers to the selected exercises marked * are given at the end of the book. A Solutions Manual and slides are available for instructors at the Web site of the book.

I assume the responsibility for all the errors in this book and would very much appreciate receiving readers' suggestions and pointing out any errors (email: d_sundararajan@yahoo.com). I am grateful to my Editor and the rest of the team at Springer for their help and encouragement in completing this project. I thank my family for their support during this endeavor.

D. Sundararajan

Contents

1	Signals	1
1.1	Basic Signals	2
1.1.1	Unit-Impulse Signal	2
1.1.2	Unit-Step Signal	4
1.1.3	Unit-Ramp Signal	5
1.1.4	Sinusoids and Complex Exponentials	5
1.2	Classification of Signals	12
1.2.1	Continuous, Discrete, and Digital Signals	13
1.2.2	Periodic and Aperiodic Signals	13
1.2.3	Even- and Odd-Symmetric Signals	14
1.2.4	Energy and Power Signals	17
1.2.5	Deterministic and Random Signals	18
1.2.6	Causal and Noncausal Signals	19
1.3	Signal Operations	19
1.3.1	Time Shifting	19
1.3.2	Time Scaling	20
1.4	Complex Numbers	22
1.5	Summary	25
2	The Discrete Fourier Transform	31
2.1	The Exponential Function	32
2.2	The Complex Exponential Function	33
2.2.1	Euler's Formula	33
2.2.2	Real Sinusoids in Terms of Complex Exponentials	34
2.3	The DFT and the IDFT	35
2.3.1	The DFT and the IDFT	39
2.3.2	The Criterion of Approximation	40
2.3.3	The Matrix Form of the DFT and IDFT	42

2.4	Applications of the DFT and the IDFT	50
2.4.1	Fourier Boundary Descriptor	50
2.5	Summary	54
3	Properties of the DFT	57
3.1	Linearity	57
3.2	Periodicity	58
3.3	Circular Time Shifting	59
3.4	Circular Frequency Shifting	59
3.5	Circular Time-Reversal	60
3.6	Duality	60
3.7	Transform of Complex Conjugates	61
3.8	Circular Convolution and Correlation	62
3.8.1	Circular Convolution of Time-Domain Sequences	62
3.8.2	Circular Convolution of Frequency-Domain Sequences	64
3.8.3	Circular Correlation of Time-Domain Sequences	66
3.9	Sum and Difference of Sequences	66
3.10	Upsampling of a Sequence	67
3.11	Zero Padding the Data	69
3.12	Symmetry Properties	71
3.13	Parseval's Theorem	74
3.14	Summary	75
4	Two-Dimensional DFT	81
4.1	Two-Dimensional DFT as Two Sets of 1-D DFTs	82
4.1.1	Computation of the 2-D DFT	84
4.2	The 2-D DFT and IDFT	96
4.3	The 2-D DFT of Real-Valued Signals	98
4.4	Properties of the 2-D DFT	99
4.5	Summary	110
5	Convolution and Correlation	113
5.1	Convolution	114
5.1.1	Linear Convolution	114
5.1.2	Circular Convolution	118
5.1.3	2-D Linear Convolution	121
5.2	Correlation	128
5.2.1	The Linear Correlation	128
5.3	Applications	130
5.3.1	Lowpass Filtering of Images	130
5.3.2	Highpass Filtering of Images	134
5.3.3	Object Detection in Images	140

- 5.3.4 Orthogonal Frequency Division Modulation 144
 - 5.3.5 Hilbert Transform 155
 - 5.4 Summary 156
- 6 Aliasing and Leakage 159**
 - 6.1 Aliasing Effect 159
 - 6.2 Leakage Effect 162
 - 6.2.1 Modeling Data Truncation 163
 - 6.2.2 Tapered Windows 164
 - 6.2.3 Hann and Hamming Windows 168
 - 6.2.4 Reducing the Spectral Leakage 171
 - 6.3 Picket-Fence Effect 174
 - 6.4 Summary 175
- 7 Fourier Series 179**
 - 7.1 Fourier Series 180
 - 7.1.1 FS as a Limiting Case of the DFT 181
 - 7.1.2 Gibbs Phenomenon 193
 - 7.2 Properties of the Fourier Series 194
 - 7.2.1 Linearity 194
 - 7.2.2 Symmetry 195
 - 7.2.3 Time Shifting 197
 - 7.2.4 Frequency Shifting 198
 - 7.2.5 Convolution in the Time Domain 198
 - 7.2.6 Convolution in the Frequency Domain 199
 - 7.2.7 Time Scaling 201
 - 7.2.8 Time Differentiation and Integration 201
 - 7.2.9 Parseval’s Theorem 203
 - 7.3 Applications of the Fourier Series 204
 - 7.3.1 Analysis of Rectified Power Supply 205
 - 7.3.2 Steady-State Response of Linear Systems 207
 - 7.4 Numerical Evaluation of the Fourier Series 210
 - 7.4.1 Aliasing Effect 212
 - 7.5 Summary 214
- 8 The Discrete-Time Fourier Transform 217**
 - 8.1 The DTFT 217
 - 8.1.1 DTFT as a Limiting Case of the DFT 219
 - 8.1.2 The DTFT of a Discrete Periodic Signal 228
 - 8.1.3 Determination of the DFT from the DTFT 228
 - 8.2 Properties of the Discrete-Time Fourier Transform 229
 - 8.2.1 Linearity 229
 - 8.2.2 Time Shifting 229
 - 8.2.3 Frequency Shifting 230

8.2.4	Convolution in the Time Domain	230
8.2.5	Correlation	231
8.2.6	Convolution in the Frequency Domain	232
8.2.7	Symmetry	233
8.2.8	Time-Reversal	234
8.2.9	Time-Expansion	235
8.2.10	Frequency-Differentiation	236
8.2.11	Summation	236
8.2.12	Parseval's Theorem and the Energy Transfer Function	238
8.3	Applications	238
8.3.1	Transfer Function and the System Response	239
8.3.2	Design of Linear-Phase FIR Digital Filters Using Windows	241
8.3.3	Digital Differentiator	242
8.3.4	Hilbert Transform	243
8.3.5	Downsampling	243
8.4	Approximation of the Discrete-Time Fourier Transform	245
8.5	Summary	246
9	The Fourier Transform	249
9.1	The FT as a Limiting Case of the FS	249
9.1.1	The FT Using Orthogonality	252
9.1.2	Existence of the FT	253
9.1.3	Determination of the FS from the FT	255
9.2	Properties of the Fourier Transform	258
9.2.1	Linearity	258
9.2.2	Duality	259
9.2.3	Symmetry	260
9.2.4	Time Shifting	261
9.2.5	Frequency Shifting	261
9.2.6	Convolution in the Time Domain	262
9.2.7	Convolution in the Frequency Domain	263
9.2.8	Conjugation	263
9.2.9	Cross-Correlation	264
9.2.10	Time-Reversal	265
9.2.11	Time Scaling	265
9.2.12	Time Differentiation	267
9.2.13	Time Integration	268
9.2.14	Frequency-Differentiation	269
9.2.15	Parseval's Theorem and the Energy Transfer Function	270

- 9.3 Fourier Transform of Mixed Class of Signals 271
 - 9.3.1 The FT of a Continuous Periodic Signal 272
 - 9.3.2 The FT of a Sampled Signal and the Aliasing Effect. 273
 - 9.3.3 The FT and the DTFT of Sampled Aperiodic Signals 277
 - 9.3.4 The FT and the DFT of Sampled Periodic Signals 277
 - 9.3.5 Reconstruction of the Continuous Signal from Its Sampled Version 280
- 9.4 Applications of the Fourier Transform 280
 - 9.4.1 Transfer Function and the System Response 280
 - 9.4.2 Ideal Filters and Their Unrealizability 283
- 9.5 Approximation of the Fourier Transform 285
- 9.6 Summary 289
- 10 Fast Computation of the DFT 293**
 - 10.1 Half-Wave Symmetry of Periodic Waveforms 294
 - 10.2 The PM DIF DFT Algorithm. 296
 - 10.3 The PM DIT DFT Algorithm 305
 - 10.3.1 Basics of the PM DIT DFT Algorithm 306
 - 10.4 Efficient Computation of the DFT of Real Data 308
 - 10.4.1 Two DFTs of Real Data Simultaneously 308
 - 10.4.2 DFT of a Single Real Data Set 312
 - 10.5 Summary 317
- Appendix A: Transform Pairs and Properties 321**
- Appendix B: Useful Mathematical Formulas 329**
- Bibliography 335**
- Answers to Selected Exercises 337**
- Index 355**

About the Author

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Over the course of his engineering career, he has held positions at the National Aerospace Laboratory, Bangalore, and the National Physical Laboratory, New Delhi, where he worked on the design of digital and analog signal processing systems.

Abbreviations

1-D	One-dimensional
2-D	Two-dimensional
DC	Sinusoid with frequency zero, constant
DFT	Discrete Fourier transform
DIF	Decimation in frequency
DIT	Decimation in time
DTFT	Discrete-time Fourier transform
FIR	Finite impulse response
FS	Fourier series
FT	Fourier transform
IDFT	Inverse discrete Fourier transform
IFT	Inverse Fourier transform
LSB	Least significant bit
LTI	Linear time-invariant
PM	Plus-minus
RDFT	DFT of real-valued data
RIDFT	IDFT of the transform of real-valued data