Classical Summability Theory

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Dedicated to my wife Vijayalakshmi and my children Sowmya and Balasubramanian

Preface

The study of convergence of infinite series is a very old art. In ancient times, people were interested in orthodox examination for convergence of infinite series. Divergent series, i.e., infinite series which do not converge, was of no interest to them until the advent of L. Euler (1707–1783), who took up a serious study of divergent series. He was later followed by a galaxy of very great mathematicians.

Study of divergent series is the foundation of summability theory. Summability theory has many utilities in analysis and applied mathematics. An engineer or physicist, who works on Fourier series, Fourier transforms or analytic continuation, can find summability theory very useful for his/her research.

In the present book, some of the contributions of the author to classical summability theory are highlighted, thereby supplementing, the material already available in standard texts on summability theory.

There are six chapters in all. The salient features of each chapter are listed below. In Chap. 1, after a very brief introduction, we recall well-known definitions and concepts. We state and prove Silverman–Toeplitz theorem, Schur's theorem and then deduce Steinhaus theorem. We introduce a sequence space Λ_r, $r \geq 1$ being a fixed integer and make a detailed study of the space Λ_r, especially from the point of view of sequences of zeros and ones. We prove a Steinhaus type result involving the space Λ_r, which improves Steinhaus theorem. We prove some more Steinhaus type theorems too.

Chapter 2 deals with the core of a sequence. We present an improvement of Sherbakhoff's result, which leads to a short and very elegant proof of Knopp's core theorem. We also present some nice properties of the class (ℓ, ℓ) of infinite matrices.

Chapter 3 is devoted to a detailed study of some special methods of summability, viz., the Abel method, the Weighted mean method, the Euler method and the (M, λ_n) or Natarajan method. We bring out the connection between the Abel method and the Natarajan method. Some product theorems involving certain summability methods are also proved.

In Chap. 4, some nicer properties of the (M, λ_n) method are established. Further, we prove a few results on the Cauchy multiplication of certain summable series.

In Chap. 5, a new definition of convergence of a double sequence and a double series is introduced. In the context of this new definition, Silverman–Toeplitz theorem for 4-dimensional infinite matrices is proved. We also prove Schur's and Steinhaus theorems for 4-dimensional infinite matrices.

Finally in Chap. 6, we introduce the Nörlund, the Weighted mean and the $(M, \lambda_{m,n})$ or Natarajan methods for double sequences and double series and study some of their properties.

I thank my mentor Prof. M.S. Rangachari for initiating me to the topic of summability theory—both classical and ultrametric. I thank Mr. E. Boopal for typing the manuscript.

Chennai, India P.N. Natarajan

Contents

About the Author

P.N. Natarajan formerly with the Department of Mathematics, Ramakrishna Mission Vivekananda College, Chennai, India, has been an independent researcher and mathematician since his retirement in 2004. He did his Ph.D. at the University of Madras, under Prof. M.S. Rangachari, former director and head of the Ramanujan Institute for Advanced Study in Mathematics, University of Madras. An active researcher, Prof. Natarajan has published over 100 research papers in several international journals like the *Proceedings of the American Mathematical Society*, *Bulletin of the London Mathematical Society*, *Indagationes Mathematicae*, *Annales Mathematiques Blaise Pascal* and *Commentationes Mathematicae* (Prace Matematyczne). His research interests include summability theory and functional analysis (both classical and ultrametric). Professor Natarajan was honored with the Dr. Radhakrishnan Award for the Best Teacher in Mathematics for the year 1990–1991 by the Government of Tamil Nadu. In addition to being invited to visit several renowned institutes in Canada, France, Holland and Greece, Prof. Natarajan has participated in several international conferences and chaired sessions. He has authored two books, *An Introduction to Ultrametric Summability Theory* and its second edition, both published with Springer in 2013 and 2015, respectively.