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Tian-You Fan

Mathematical Theory of Elasticity of Quasicrystals and Its Applications

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Preface

The first edition of this book was published by Scienc Press, Beijing/Springer-Verlag, Heidelberg, in 2010 mainly concerning a mathematical theory of elasticity of solid quasicrystals, in which the Landau symmetry breaking and elementary excitation principle plays a central role. Bak, Lubensky and other pioneering researchers introduced a new elementary excitation—phason drawn from theory of incommensurate phase apart from the phonon elementary excitation well-known in condensed matter physics.

Since 2004, the soft-matter quasicrystals with 12-fold symmetry have been observed in liquid crystals, colloids and polymers; in particular, 18-fold symmetry quasicrystals were observed in 2011 in colloids; this symmetry in quasicrystals is discovered for the first time. These observations belong to an important event of chemistry in twenty-first century and have attracted a great deal of attention of researchers. Readers are interested in many topics of the new area of study. However, accumulated experimental data related with mechanical behaviour of the new phase are very limited, there is the lack of fundamental data, the mechanism of deformation and motion of the matter has not sufficiently been explored after the discovery over one decade, and it leads to fundamental difficulties to the study. Due to these difficulties, an introduction to soft-matter quasicrystals is given in very brief in Major Appendix of this book.

Though the new edition increases some new contents, the title of the book has not been changed, because the main part of which is still concerned with elasticity of solid quasicrystals, and only new chapter—Chap. 16—on hydrodynamics of quasicrystals is added; the introduction on soft-matter quasicrystals is very limited and listed in the Major Appendix. The changes of the contents of the first 15 chapters are not too great; we add some examples with application significance and exclude ones of less practical meaning; a part of contents of Appendix A is moved into the Appendix of Chap. 11, and add a new appendix, i.e. Appendix C in the Major Appendix, in which some additional derivations of hydrodynamic equations of solid quasicrystals based on the Poisson bracket method are included, which may be referred by readers. Some type and typesetting errors and mistakes contained in

the first edition are removed, but some new errors and mistakes might appear in the new edition; any criticisms from readers are warmly welcome!

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Beijing, China

Tian-You Fan

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Notations

\mathbf{r}	Radius vector
Ω	Domain
S	Boundary of domain
S_u	Boundary part at which the displacements are given
S_t	Boundary part at which the tractions are given (or S_σ at which the applied stresses are given)
ρ	Mass density (g/cm^3)
p	Fluid pressure ($\text{Pa} = \text{N/m}^2$)
\mathbf{u}	Phonon-type displacement field (cm)
\mathbf{w}	Phason-type displacement field (or second phason displacement field only for quasicrystals with 18-fold symmetry) (cm)
\mathbf{V}	Velocity field of solid viscosity (cm/s)
$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$	Phonon strain tensor
$w_{ij} = \frac{\partial w_i}{\partial x_j}$	Phason strain tensor (or second phason strain tensor only for quasicrystals with 18-fold symmetry)
$\dot{\zeta}_{ij} = \frac{1}{2} \left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \right)$	Deformation velocity tensor of solid viscosity (1/s)
σ_{ij}	Phonon stress tensor (Pa)
H_{ij}	Phason stress tensor (or second phason stress tensor only for quasicrystals with 18-fold symmetry) (Pa)
σ'_{ij}	Viscosity stress tensor (Pa)
C_{ijkl}	Phonon elastic coefficient tensor (Pa)
K_{ijkl}	Phason elastic coefficient tensor (or second kind phason elastic coefficient tensor only for quasicrystals with 18-fold symmetry) (Pa)
R_{ijkl}	Phonon-phason coupling elastic coefficient tensor (u - w coupling elastic coefficient tensor) (Pa)
η	First viscosity coefficient of fluid ($0.1 \text{ Pa} \cdot \text{s} = \text{Poise}$)
η/ρ	First kinetic viscosity coefficient of fluid (cm^2/s)

ζ	Second viscosity coefficient of fluid (0.1 Pa · s = Poise)
ζ/ρ	Second kinetic viscosity coefficient of fluid (cm ² /s)
Γ_u	Phonon dissipation coefficient (m ³ s/kg)
Γ_w	Phason dissipation coefficient (or second kind phason dissipation coefficient tensor only for quasicrystals with 18-fold symmetry) (m ³ s/kg)
\mathbf{v}	First phason-type displacement field (only for quasicrystals with 18-fold symmetry) (cm)
$v_{ij} = \frac{\partial v_i}{\partial x_j}$	First phason strain tensor (only for quasicrystals with 18-fold symmetry)
τ_{ij}	First phason stress tensor (only for quasicrystals with 18-fold symmetry) (Pa)
r_{ijkl}	Phonon-first phason coupling elastic coefficient tensor (or u - v coupling elastic coefficient tensor only for quasicrystals with 18-fold symmetry) (Pa)