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Recent Developments in Anisotropic Heterogeneous Shell Theory

Applications of Refined and
Three-dimensional Theory—Volume IIB

 Springer

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From the Preface of Book 1

The theory of shells is an independent and highly developed science, logically based on the theory of elasticity. Constructions consisting of thin-walled elements, have found widespread application in mechanical engineering, civil and industrial construction, ships, planes and rockets building, as well as transport systems. The development of different shell models requires the application of hypotheses based on elasticity theory leading to a reduction in terms of two-dimensional equations that describe the deformation of the shell's middle surface. The solution of shell problems requires use of various numerical methods and involves great difficulties of computational nature. The authors present discrete-continuum approaches which they developed for solving problems of elasticity theory and which allow to reduce the initial problem to systems of ordinary differential equations. These are then solved by the stable numerical method of discrete orthogonalization and will be presented in this book. On the basis of these approaches a solution for a wide class of problems of stationary deformation of anisotropic heterogeneous shells is obtained.

The monograph consists of two books, each of which consists of three chapters. A summary of the chapters is as follows.

Chapter 1 (Volume IIA): The solutions of stress-strain problems for a wide class of anisotropic inhomogeneous shells obtained by the refined model are presented. Studying these problems results in difficult calculations due to partial differential equations with variable coefficients. For solving the problem, spline-collocation and discrete-orthogonalization methods are used. The influence of geometrical and mechanical parameters, of the boundary conditions, and of the loading character on the distributions of stress and displacement fields in shallow, spherical, conical, and noncircular cylindrical shells is analyzed. The dependence of the stress-strain pattern on shell thickness variation is studied. The problem was solved also in the case of the thickness varying in two directions. It is studied how the rule of variation in the thickness of the shells influences their stress-strain state. Noncircular cylindrical shells with elliptical and corrugated sections are considered.

The results obtained in numerous calculations support the efficiency of the discrete-orthogonalization approach proposed in the monograph for solving static problems for anisotropic inhomogeneous shells when using the refined model.

Chapter 1 (Volume IIB): A wide class of problems of natural vibrations of anisotropic inhomogeneous shells is solved by using a refined model. Shells with constructional (variable thickness) and structural inhomogeneity (made of functionally gradient materials) are considered. The initial boundary-value eigenvalue partial derivative problems with variable coefficients are solved by spline-collocation, discrete-orthogonalization, and incremental search methods. In the case of hinged shells, the results obtained by making use of analytical and proposed numerical methods are compared and analyzed. It is studied how the geometrical and mechanical parameters as well as the type of boundary conditions influence the distribution of dynamical characteristics of the shells under consideration. The frequencies and modes of natural vibrations of an orthotropic shallow shell of double curvature with variable thickness and various values of curvature radius are determined. For the example of cylindrical shells made of a functionally gradient material, the dynamical characteristics have been calculated with the thickness being differently varied in circumferential direction. The values of natural frequencies obtained for this class of shells under some boundary conditions are compared with the data calculated by the three-dimensional theory of elasticity.

Chapter 2 (Volume IIB): The model of the three-dimensional theory of elasticity is employed in order to study stationary deformation of hollow anisotropic inhomogeneous cylinders of finite length. Solutions of problems of the stress-strain state and natural vibrations of hollow inhomogeneous finite-length cylinders are presented, which were obtained by making use of spline-collocation and discrete-orthogonalization methods. The influence of geometrical and mechanical parameters, of boundary conditions, and of the loading character on distributions of stress and displacement fields, as well as of dynamical characteristics in the above cylinders is analyzed. For some cases the results obtained by three-dimensional and shell theories are compared. When solving dynamical problems for orthotropic hollow cylinders with different boundary conditions at the ends, the method of straight-line methods in combination with the discrete-orthogonalization method was also applied. Computations for solid anisotropic finite-length cylinders with different end conditions were carried out by using the semi-analytical finite element method. In the case of free ends the results of calculations the natural frequencies were compared with those determined experimentally. The results of calculations of mechanical behavior of anisotropic inhomogeneous circular cylinders demonstrate the efficiency of the discrete-continual approaches proposed in the monograph for solving shell problems using the three-dimensional model of the theory of elasticity.

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