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# Bayesian Methods in the Search for MH370

 Springer Open

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ISSN 2191-8112                      ISSN 2191-8120 (electronic)  
SpringerBriefs in Electrical and Computer Engineering  
ISSN 2196-4076                      ISSN 2196-4084 (electronic)  
SpringerBriefs in Signal Processing  
ISBN 978-981-10-0378-3            ISBN 978-981-10-0379-0 (eBook)  
DOI 10.1007/978-981-10-0379-0

Library of Congress Control Number: 2015960765

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The registered company is Springer Science+Business Media Singapore Pte Ltd.

# Foreword I

Uncertainty is all pervasive—whether it relates to everyday personal choices and actions, or as background to business and policy decisions, or economic and climate predictions. In recent times, few things have attracted as much attention as the uncertainty surrounding the final whereabouts of MH370.

How to deal scientifically with uncertainty? Put simply, on the one hand there are events or outcomes of interest that we don't know; on the other hand, pieces of information that we judge relevant in some sense that we do know. We need to assess what we believe about the unknowns, given the knowns.

Formalising our measure of uncertainty in terms of probabilities, the scientific approach is encapsulated in the so-called Bayesian statistical paradigm, in which beliefs about the unknowns are quantified by a probability measure conditional on what we know.

But typically, our state of knowledge itself gets modified over time and a method is therefore needed to refine and update beliefs as new information is acquired and assembled. The logical, mathematical rule for carrying out this updating is Bayes theorem, hence the term Bayesian Methods to describe the analytic and computational toolkit that has been developed for updating beliefs as evidence changes or is added to.

It is this toolkit that has been employed in the search for MH370 and this fascinating book provides a blow-by-blow case account of how the various strands of evidence have been brought together to give an overall probabilistic assessment of the final whereabouts of the plane.

This has been an extremely complex task and the authors are to be congratulated on setting out systematically and coherently the science and mathematics driving the evidential equations. But, in addition to the complex modelling there remains the task of pulling out the Bayesian probability messages from the tangle of data that has been assembled. The computational methods for achieving this are of relatively recent origin and, on a personal note, I am delighted to have played a small part, with Dr. Neil Gordon, in the signal processing revolution that is now the particle filter method of analysis.

Dr. Gordon and his colleagues have employed these methods to provide a marvellous case study demonstrating the power of mathematical modelling and computation to attack one of the most intractable uncertain puzzles of recent times.

London, UK  
November 2015

Professor Sir Adrian Smith  
FRS, Vice Chancellor  
University of London

# Foreword II

The disappearance of the Malaysian Airlines flight MH370 from air traffic control radar on the evening of 8 March 2014 with 227 passengers and 12 crew on board was a tragedy and remains a mystery until we are able to locate the flight recorder or the wreckage. Only then can we unravel what actually occurred.

Initially, the multinational search and recovery effort was focused on the Gulf of Thailand and the South China Sea. However, following closer analysis of Inmarsat data and flight path projections the search shifted to the Indian Ocean.

The Malaysian government accepted the Australian government's offer to take the lead in the search and recovery operation in the southern Indian Ocean. Australia, led by the Australian Transport Safety Bureau (ATSB), took responsibility for defining the underwater search area in 28 April 2014.

The Defence Science and Technology (DST) Group, being the only Australian government agency with the combined knowledge and experience in the scientific disciplines to support the search for MH370, became involved from May 2014. Defence scientists contributed a range of expertise across a spectrum of technologies to the search: underwater acoustics, satellite communication systems and statistical data processing.

DST Group's position as a trusted government adviser with stewardship of the full range of defence technologies has been critical to our ability to contribute to the multi-agency—and multinational—search. Our ability to provide high quality, internationally respected, rapid response is built on our deep foundational research capability. Our support to the MH370 incident is an example of scientific research feeding directly into an active operational search.

The quest for the MH370 demonstrates our ability to 'work the full problem' with a team of internationally-recognised experts contributing to the ATSB working group. Indeed, DST Group is proud to have been able to contribute its world-renowned expertise to the ATSB-led search. However, this book is not about the search for MH370. Rather, it focuses on the work to define the search zone.

We are particularly fortunate to have Dr. Neil Gordon leading the DST Group team. Dr. Gordon is recognised internationally as an expert in statistical data

processing and, in particular, the dynamic Bayesian estimation methods deployed in this activity.

Working alongside Dr. Gordon are other Defence scientists—Dr. Samuel Davey, Dr. Mark Rutten and Dr. Jason Williams who are all experts in target tracking and multi-sensor fusion, and Dr. Ian Holland who specialises in satellite and wireless communications.

DST Group’s expertise and unique capabilities will continue to contribute to the ongoing search for the MH370, led by the ATSB and in collaboration with other Australian and international agencies.

Canberra, Australia  
November 2015

Dr. Alex Zelinsky  
Chief Defence Scientist  
Defence Science and Technology  
Department of Defence



# Acknowledgements

This book is a summary of our work as members of the MH370 Flight Path Reconstruction group. The group comprised members from Inmarsat, Thales, Boeing, US National Transportation Safety Board (NTSB) and the UK Air Accidents Investigation Branch (AAIB). The group has been expertly led by the Australian Transport Safety Bureau (ATSB).

To all our friends and colleagues at the ATSB: During many intense and stressful times you always remained calm and assured. You are dedicated professionals.

To the Flight Path reconstruction working group: We thank you all for freely sharing your expert knowledge and for working together both independently and collaboratively in such a positive way. We also thank you for sharing your forth-right opinions and views and questioning everything.

To our friends and colleagues at the Defence Science and Technology Group: We would particularly like to thank Gerald Bolding and Balachander Ramamurthy for their support to the satellite data analysis and David Liebing for all his efforts in drawing the team together.

The material in Chap. 11 related to the Reunion Island debris find was developed with David Griffin from CSIRO Marine and Atmospheric Research in Hobart. Thank you David for hosting us and sharing your knowledge.

We thank Sanjeev Arulampalam (DST Group, Australia), Vaughan Clarkson (University of Queensland, Australia), Simon Godsill (Cambridge University, UK), Fredrik Gustafsson (Linköping University, Sweden), Simon Maskell (Liverpool University, UK), and Thomas Schön (Uppsala University, Sweden) for their thoughtful and helpful comments and suggestions. We thank Brian Anderson for communicating some minor corrections to the presentation.

All those involved in the search for MH370 remain totally committed to finding the aircraft and helping find closure for the families involved.

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**Neil Gordon** received a Ph.D. in Statistics from Imperial College London in 1993. He was with the Defence Evaluation and Research Agency in the UK until 2002 working on missile guidance and statistical data processing. He is best known for initiating the particle filter approach to nonlinear, non-Gaussian dynamic estimation, which is now in widespread use throughout the world in many diverse disciplines. He is the co-author/co-editor of two books on particle filtering. In 2002 he moved to the Defence Science and Technology Group in Adelaide, Australia, where he is currently head of Data and Information Fusion. In 2014 he became an Honorary Professor with the School of Information Technology and Electrical Engineering at the University of Queensland. He is a Senior Member of the IEEE.

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# Acronyms

|           |  |
|-----------|--|
| 9M-MRO    | Registration number of the accident aircraft                                       |
| AAIB      | Air Accidents Investigation Branch (United Kingdom)                                |
| ACARS     | Aircraft Communications Addressing and Reporting System                            |
| ACCESS-G  | Australian Community Climate and Earth-System Simulator Global model               |
| AES       | Aircraft Earth Station (the aircraft satellite communications unit)                |
| ATC       | Air Traffic Control  |
| ATSB      | Australian Transport Safety Bureau   |
| BFO       | Burst Frequency Offset   |
| BTO       | Burst Time Offset  |
| cdf       | Cumulative Distribution Function   |
| CI        | Cost Index (used to automatically control air speed)                               |
| CMH       | Constant Magnetic Heading navigation   |
| CMT       | Constant Magnetic Track navigation   |
| CSIRO     | Commonwealth Scientific and Industrial Research Organisation                       |
| CTH       | Constant True Heading navigation   |
| CTT       | Constant True Track navigation   |
| DST Group | Defence Science and Technology Group, Australia                                    |
| GES       | Ground Earth Station (the ground component of the satellite communications system) |
| HPD       | Highest Posterior Density  |
| JACC      | Joint Agency Coordination Centre   |
| KL        | Kuala Lumpur   |
| LNAV      | Lateral NAVigation   |
| NEES      | Normalised Estimation Error Squared  |
| NTSB      | National Transportation Safety Board (United States)                               |
| OU        | Ornstein–Uhlenbeck process   |

|        |  |
|--------|--|
| pdf    | Probability Density Function               |
| RMS    | Root Mean Squared                          |
| SATCOM | Satellite communications                   |
| SIR    | Sample Importance Resample particle filter |
| UTC    | Coordinated Universal Time                 |