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Linguistic Values Based Intelligent Information Processing: Theory, Methods, and Applications

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Preface

Humans employ mostly natural languages in describing and representing problems, computing and reasoning, arriving at final conclusions described similarly as words in a natural language or as the form of mental perceptions. To make machines imitate humans' mental activities, the key point in terms of machine intelligence is to process uncertain information by means of natural languages with vague and imprecise concepts.

Zadeh (1996a) proposed a concept of Computing with Words (CWW) to model and compute with linguistic descriptions that are propositions drawn from a natural language. CWW, followed the concept of linguistic variables (Zadeh, 1975a,b) and fuzzy sets (Zadeh, 1965), has been developed intensively and opened several new vast research fields as well as applied in various areas, particularly in the area of artificial intelligence.

Zadeh (1997, 2005) emphasized that the core conceptions in CWW are linguistic variables and fuzzy logic (or approximate reasoning). In a linguistic variable, each linguistic value is explained by a fuzzy set (also called semantics of the linguistic value), its membership function is defined on the universe of discourse of the linguistic variable. By fuzzy sets, linguistic information or statements are quantified by membership functions, and information propagation is performed by approximate reasoning. The use of linguistic variables implies processes of CWW such as their fusion, aggregation, and comparison. Different computational approaches in the literature addressed those processes (Wang, 2001; Zadeh and Kacprzyk, 1999a, b). Membership functions are generally at the core of many fuzzy-set theories based CWW.

Apart from fuzzy-set theories based CWW, there exist some alternative methods developed to model and compute with linguistic information in natural languages from the different point of view. This book provides a systematic introduction course of those alternative methods to modify and overcome limitations of CWW in the sense of Zadeh, *e.g.*, diffi-

culty in determining and interpreting fuzzy set membership functions of linguistic values, computational complexity and loss of information due to linguistic approximations. Both of these steps usually imply the investigation of human factor, semantics of the linguistic terms, and subjective beliefs. In this book, we introduce the so-called *linguistic-valued based intelligent information process approach* – some popular approaches in which the use of membership functions associated to the linguistic terms is unnecessary or the membership function does not play an important or key role. The linguistic value reflects the use of “words” as computational variables, *i.e.*, the direct use of linguistic values in natural language, where the symbolic approach acts by the direct computation and reasoning on linguistic terms. Although some work introduced in this book also cover the use of membership function to investigate the semantics of linguistic variables in different alternative ways, however, a key insight and the main focus behind and within this book is to directly represent and manipulate the available linguistic information and knowledge from the ordinal point of view, the algebraic point of view or from the symbolic logic point of view.

Taking a wider perspective these theories, methods and algorithms will benefit researchers in a very wide variety of domains but in particular: AI and decision science, and especially, in social science and political science often involved in human judgement, reasoning and decision making with linguistic evaluation in natural language. The linguistic formal method in this research direction naturally lends to an effective means of communication between scholars and practitioners, since the approach greatly facilitates the exchange and analysis of “raw” linguistic information. Carrying out such a research will probably not be an easy task, but it is nonetheless worth pursuing.

This book serves a major reference for scientists and engineers interested in applying new fuzzy logic approach tools to achieve intelligent solution in complex systems. It can be also used in special courses of fuzzy logic and artificial intelligence, for researchers and graduate students, in advance courses on applications of AI and decision support systems. The book thus addresses a rather broad public: logicians, linguists, philosophers as well as theoretical computer scientists and mathematicians, engineers with interests in linguistic information processing and their applications.

The book has six chapters.

Chapter 1 provides the academic background of the development of linguistic valued based intelligent information process, overviews and classifies theories and methods in this direc-

tion, also reviews basic algebraic concepts necessary for characterization of the linguistic values and for understanding the subsequent chapters, finally outlines the potential applications.

Chapter 2 starts with a discussion of general linguistic decision analysis framework, followed with one representative linguistic valued approach, called the fuzzy ordinal linguistic approach, a variety of different candidates within this framework especially in terms of aggregation operators will be reviewed. The main focus is given on the 2-tuple fuzzy linguistic representation model as one of most popular fuzzy ordinal linguistic approaches. Formally, the academic idea of fuzzy ordinal linguistic approach is that the finite set of linguistic values is embedded in a linear ordered structure. Hence, every linguistic value can be identified by a number, which serves as an index of the linguistic value. The advantages of fuzzy ordinal linguistic approach are membership functions associated to the linguistic values are unnecessary in the computation process. Furthermore, they are computationally simple and quick. The 2-tuple fuzzy linguistic representation model inherits those advantages, and provides a better representation and computation scheme to avoid the loss of information during the normal fuzzy ordinal linguistic approach. A summary introduction of the 2-tuple fuzzy linguistic representation model is given and some of its applications are also provided to help readers understand the 2-tuple fuzzy linguistic representation model.

Chapter 3 introduces hedge algebras of linguistic values, in which linguistic representation and manipulation are mainly investigated from the algebraic point of view. The academic idea of hedge algebras is that the set of linguistic values is embedded in a natural and rich enough algebraic structure with a general partially ordering determined by natural meaning of linguistic terms, a basic structure of universal algebras. Formally, by analyzing linguistic hedges, one can discover an ordering relation on the set of linguistic values, and the ordering relation is based on intuitive meaning of linguistic values. Therefore, the motivation and focus of the researchers in this direction are to discover an algebraic structure of terms-domains of linguistic variables in the category of universal algebras. The chapter summarizes hedge algebras, including those specialized version, *e.g.*, symmetrical hedge algebras and complete hedge algebras. Meanwhile, hedge algebras are applied in linguistic reasoning, constructing membership functions of linguistic values and fuzzy control.

Chapter 4 introduces linguistic-valued information processing mainly based on a logical algebra structure - lattice implication algebras (LIA). The chapter starts from the general introduction of LIA and its corresponding logical system and approximate reasoning framework. The linguistic truth-valued lattice implication algebra is detailed afterwards. A key

insight behind the linguistic-valued logic scheme is that we can use natural language to express a logic in which the truth values of propositions are expressed as linguistic values in natural language terms such as *true*, *very true*, *less true*, *very false*, and *false*, instead of a numerical scale, such an approach could reduce approximation errors that could estimate membership functions and could also treat vague information in its true format. Specially, the set of linguistic truth values is embedded in LIA, which is inspired from hedge algebra, established by analyzing semantic heredity of linguistic hedges. Based on the extensive work on the LIA based logical system and reasoning approaches, the main research has been focused on studying linguistic representation and manipulation from the logical point of view. Besides the LIA based linguistic-valued algebra, linguistic truth-valued propositional logic, linguistic truth-valued automated reasoning theory and approaches, linguistic truth-valued approximate reasoning, and relevant applications in decision making, knowledge-based system and evaluation are also summarized.

Chapter 5 introduces fuzzy number indexes of linguistic values. The places of linguistic values in the ordered structure of linguistic values are represented by fuzzy numbers. However, it differs from the fuzzy set of the linguistic value, which expresses the membership degrees of objects belonging to the linguistic value. The advantages of fuzzy number indexes of linguistic values are that (1) incomparable linguistic values are identified by their fuzzy number indexes; (2) deciding fuzzy number indexes of linguistic values is based on intuitive orders of linguistic values given by individuals. Hence, the universe of discourse of linguistic values is not necessary; (3) incomparable linguistic information processing is easily transformed into numerical calculus. After a brief introduction of the approach, some applications on establishing new aggregation operators in decision making, fuzzy risk analysis, as well as information gathering in multi-agent systems are also highlighted.

Chapter 6 introduces the hierarchical structure analysis of linguistic values. The main focus is on a new framework for linguistic modeling. Formally, for each object of the domain, there is a common ordering of linguistic values shared by all individuals. Linguistic labels are assumed to be chosen from a finite predefined set of labels and the set of appropriate labels for a value is defined as a random set from a population of individuals into the set of subsets of labels. Then the appropriateness degree, in contrast to a membership degree, of a value to a label is derived from that probability distribution, or mass assignment, on the set of label subsets. The framework also provides a coherent calculus for linguistic expressions composed by logical connectives on linguistic labels. Moreover, this helps to construct the formal context of linguistic values, obtain linguistic formal concepts and analyze the

relation and hierarchical structure among linguistic values. The appropriateness degrees of linguistic values, the formal context of linguistic values and the hierarchical structure analysis of linguistic values based on formal concept analysis are also summarized.

Chapter 7 concludes the book and lists many references related to linguistic values based intelligent information processing. We hope the book is useful for readers to get a clear picture of the newly presented theories, methods, applications and future research direction about linguistic values based intelligent information processing.

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