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Volume 1

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Astroparticle Physics: Theory and Phenomenology

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I dedicate this monograph to all colleagues who played a significant role in my scientific life and, last but not least, to my parents who to a large part provided the support and conditions necessary to have the freedom to embark on a scientific career.

Foreword

Astroparticle physics occupies a peculiar place among modern branches of fundamental sciences. At a time of great specialization and very advanced technical tools in astrophysics, cosmology and particle physics, it finds its *raison d' être* in the bridges built between these disciplines and in its peculiar interdisciplinary approach. It deals with some of the most unusual and extreme systems to be found in Nature: not surprisingly, it may thus involve mechanisms at the edge of current knowledge, occasionally shedding light on processes beyond current “standard models”, as recently happened for neutrino mixing. However, Nature does not care about human subdivisions of disciplines: in the core collapse of a supernova, for instance, gravity, (magneto)fluidodynamics, microscopic neutrino physics, and the equation of state of dense matter all play a crucial role. Similarly, in many processes of interests in high-energy astrophysics, electromagnetic phenomena and high-energy physics processes are equally important.

Here lies the blessing of the great fascination of this field as well as the curse of mastering it, notably for students and beginning practitioners. The associated difficulties also reflect into the limited number of reference textbooks available, if compared with disciplines with long traditions and a narrower but more systematic content. Günter Sigl, a scholar who has distinguished himself in many branches of astroparticle physics, has now tackled the challenge of writing a very comprehensive monograph. I am amazed by the breadth of the result, covering from introductory notions (such as the particle physics, fluid dynamics or plasma astrophysics ones) to hot topics of current research, including for example the gravitational waves signal from black hole mergers recently observed by LIGO or the latest news on the still mysterious fast radio bursts. Here and there, one also recognizes the *personal touch* reserved by G. Sigl to some of his beloved topics, such as his general kinetic description of mixed flavor states scattering in a medium, or the detailed treatment of propagation effects of energetic particles in the extragalactic space. Via different paths along its five parts and sixteen chapters, this book is suitable to both students who start a career in astroparticle physics and as a reference work for accomplished researchers.

It is customary to wish for a newly edited book that it would remain as a reference for many years to come. While I am sure that most parts of this one will, a rapid evolution of several subjects covered will be the most tangible measure of its success among young generations and of its impact on the overall progress of the discipline. To the many more revised editions to follow, then!

Annecy-le-Vieux
August 2016

Pasquale Dario Serpico

Preface

This monograph grew out of a series of lectures and seminars on cosmology, astroparticle physics and general relativity I have given or co-organized at University of Hamburg and DESY. At the same time my group at University of Hamburg has worked on a range of topics in these areas so that the need for a comprehensive reference for both teaching and as a basis of new research projects arose that would also show the connections between the different subfields wherever possible. During the work on this book project I encountered many more interesting links than I would ever have imagined. For example, the concept of topologically non-trivial field configurations such as sphalerons, well known in particle physics, have a counterpart in plasma physics and magnetohydrodynamics known as magnetic helicity which can dramatically influence the evolution of primordial magnetic fields or magnetic fields in compact stars. The sphalerons of particle physics can even directly give rise to magnetic helicity on macroscopic scales which can have consequences in situations as seemingly far away as the propagation of electromagnetic cascades. Another fascinating example is the interplay between coherent quantum oscillations between states of different “flavors” but with the same momentum such as in neutrino oscillations and the interactions between different momentum modes which can lead to the decoherence of such oscillations and even to so far little understood instabilities. Furthermore, the techniques used to derive the equations describing these phenomena, such as the “non-abelian Boltzmann equations” describing the oscillations of neutrinos including their self-interactions, can be applied to completely different domains such as the evolution of the power spectra of magnetic fields and velocity flows in the Gaussian closure approximation where the fields are assumed to have only two-point correlations that are homogeneous and isotropic, which is equivalent to assuming that different wavenumber modes are uncorrelated. For the description of scattering of (possibly multi-flavor) states by Boltzmann equations this corresponds to the assumption of “molecular chaos” and it has very interesting connections to Boltzmann’s H theorem and the second law of thermodynamics that I have tried to describe.

At the same time, by trying to dig deeper into some of the open questions it occurred to me how little sometimes is still understood, for example, the

propagation of charged particles in turbulent magnetic fields or whether instabilities can occur in magnetic fields in the presence of chiral asymmetries and which consequences this could have in the early Universe or in core collapse supernovae. Motivated by getting aware of such open questions, I devoted significant parts of this book to a description of the open problems surrounding some such poorly understood phenomena.

As a result, this book on the one hand tries to provide a solid base of well-established facts and summarize current knowledge in various subfields of astroparticle physics. On the other hand, it also attempts to show current limitations of our knowledge including some selected speculative ideas. I hope this could be useful in particular for young researchers considering to enter the field of astroparticle physics theory and phenomenology by both providing a solid basis and conveying excitement for some of the unresolved open questions in this field.

The book is structured as follows: The first part recapitulates the necessary background in particle physics. It consists of two chapters: Chap. 1 reviews Lorentz transformations and the Dirac equation, gives a mostly phenomenological description of the electroweak interactions, in particular neutrino physics, and prepares the discussion of their role in astrophysics and cosmology in particular in Part VI of the book. Chapter 2 describes the fundamental interactions from a more general perspective, starting with the general principles and subsequently describing all four fundamental interactions, namely the electromagnetic, electroweak, strong and gravitational interactions, in some detail. This also includes quantum field theory effects such as anomalies which can play an important role in early Universe cosmology and even in magnetohydrodynamics, as mentioned above, and will be taken up in later chapters. Chapter 2 ends with a brief discussion of extensions of the Standard Model, including theories of a Grand Unification of the non-gravitational interactions, supersymmetry, extra dimensions and string theory.

In the second part of the book we summarize the fundamentals of cosmology and astrophysics in roughly reverse chronological order starting from today's Universe and going back in time all the way to the big bang. Chapters 3 and 4 are roughly divided by the time of decoupling of photons which roughly coincides with the formation of the cosmic microwave background. Thus, Chap. 3 describes the "late" Universe between redshift $z = 0$ and $z \simeq 1100$, starting with a discussion of astrophysical sources powered by nuclear energy, i.e. stars, and by accretion, in particular by massive black holes, continuing with a description of the expanding Universe and the formation of structure, and ending with an overview of the role of plasma physics and magnetohydrodynamics in astrophysics. Chapter 3 closes with a brief discussion of gravitational lensing which plays an important role in modern cosmological observations. Chapter 4 starts with a recapitulation of basic facts of equilibrium thermodynamics that will be necessary in later chapters. It continues with an overview over the nowadays well-established physics of the cosmic microwave background, the role of freeze-out from thermal equilibrium for neutrinos and dark matter, and primordial nucleosynthesis. The subsequent sections of Chap. 4 cover the earliest parts of the history of our Universe which necessarily are less well understood, starting with phase transitions and possible relics they

could leave behind, coming back to magnetohydrodynamics and, this time, its role in creating the first seeds of primordial magnetic fields, continuing with the origin of ordinary matter via the genesis of leptons and baryons, and with the origin of density perturbations, and thus eventually structure, in inflationary scenarios in which the Universe underwent a period of exponential expansion which also provides solutions to some well known cosmological problems such as causality, curvature and the abundance of certain topological defects that would otherwise be over-produced in phase transitions discussed before, and ending with some more speculative thoughts on cosmological initial conditions and the arrow of time.

The remaining parts of this monograph are roughly ordered in increasing “elusiveness” of the objects involved. Consequently, Part III is concerned with the strongly and electromagnetically interacting messengers of our Universe, namely charged cosmic rays and high-energy photons. Part IV covers the physics of weakly coupled messengers within the Standard Model, namely neutrinos and gravitational waves. The last part, Part V, is finally devoted to dark matter and various other possible phenomena beyond the Standard Model such as new light matter states including axion-like particles and hidden photons, and the role of possible violations of the Lorentz symmetry.

The subject of Part III is the physics and astrophysics of high-energy charge cosmic rays and γ -rays between MeV energies and the highest ever observed energies above 10^{20} eV. These particles are messengers and witnesses of some of the most violent processes in the Universe. Part III is divided into four chapters. Chapter 5 covers the various detection techniques and their physical foundation, in particular the relevant physics of particle propagation and shower development in media, as well as an overview of the current experimental results. This includes a discussion of aspects of air shower physics that are not yet fully understood such as the muon content and its relation to the primary particle composition. Chapter 6 then presents cosmic ray acceleration mechanisms and source models, in particular active galactic nuclei and γ -ray bursts. Cosmic ray acceleration has again strong links to plasma physics and magnetohydrodynamics which continues into the next chapter on cosmic ray propagation. Chapter 7 is divided into the propagation of Galactic and extragalactic cosmic rays. Galactic cosmic rays hardly interact during their diffusion in the Galactic magnetic fields and the main complication is the theoretical description of this diffusive motion which is still surprisingly little understood and to which a long section is dedicated. For extragalactic cosmic rays inelastic interactions such as the production of pions and electron positron pairs play a prominent role, but also the semi-ballistic or, at energies below $\simeq 10^{17}$ eV, diffusive motion in the poorly known extragalactic magnetic fields represents an important ingredient. The final Chap. 8 of Part III is devoted to γ -ray astrophysics. It starts with a detailed section describing the principal electromagnetic processes and ends with an overview of Galactic and extragalactic γ -ray sources.

Part IV is devoted to two messengers that are much more difficult to detect due to their small interaction cross sections with ordinary matter, but at the same time and for the same reason can be used to “look” much deeper into the workings of the

most interesting sources and processes in the Universe, namely neutrinos and gravitational waves. The first four chapters of Part IV covers the role of neutrinos in astrophysics and cosmology. Chapter 9 discusses high-energy neutrinos. It starts with a discussion of the charged and neutral current interactions relevant for detection, turns to the production of neutrinos by hadronic interactions within cosmic ray sources, and finishes with the neutrinos produced during the propagation of primary cosmic rays from their source to the observer. Various aspects of the recent first detection of high-energy extraterrestrial neutrinos by the IceCube experiment are also discussed in this chapter. Chapter 10 is devoted to neutrino properties, in particular neutrino mixing and open questions concerning their nature such as whether they are identical to their antiparticles or not. Chapter 11 then presents the physics of neutrino oscillations in some detail. On the experimental side the observation of neutrino oscillations in laboratory and long baseline experiments is briefly discussed. On the theoretical side the role of matter effects and self-interactions and their description in terms of non-abelian Boltzmann equations is developed in some detail. The subject of Chap. 12 is the role of neutrino properties and their role in astrophysics and cosmology. This includes solar and atmospheric neutrino oscillations, the role of neutrinos in supernovae, as hot dark matter and their role in the genesis of leptons and baryons in the early Universe. Finally, Chap. 13 covers gravitational waves. After a short introduction to their theoretical description, based on the section on the gravitational interaction in Chap. 2, an overview of detection techniques is given, followed by a discussion of various astrophysical sources of gravitational waves and of primordial gravitational waves. It finishes with a short section on the prospects of multi-messenger studies based on gravitational waves, cosmic rays, γ -rays and neutrinos.

The final part, Part V, of this book focuses on possible tests of new physics beyond the Standard Model in the context of the yet unknown nature of dark matter, possible new light particles and possible violations of the Lorentz symmetry. Chapter 14 covers most relevant aspects of dark matter. It starts with a summary of the existing astrophysical and cosmological evidence for its existence. The ensuing sections then cover direct dark matter detection experiments in the laboratory, signatures of dark matter in accelerator experiments and indirect detection strategies which exploit the final state products of dark matter annihilation or decay in astrophysical objects ranging from stars, our Galaxy to other galaxies and clusters of galaxies. Chapter 15 discusses possible new light and heavy matter states that on a fundamental level are partly motivated by the CP conserving character of the strong interaction, by string theory and by other physics beyond the Standard Model. Such particles include axion-like particles (ALPs) and hidden photons and other weakly interacting sub-eV particles (WISPs), as well as heavy topological and non-topological objects such as magnetic monopoles and Q-balls. After discussing the relevant particle physics, the possible astrophysical and cosmological roles played by these particles are discussed, including their possible contribution to the dark matter. Finally, the subject of Chap. 16 is a discussion of the possible effects of small violations of fundamental symmetries such as the Lorentz symmetry or the equivalence principle which is also often motivated by string theory. It contains a

section on the role of such effects at very high energies where they could modify the propagation of cosmic rays, and on their role at low energies. A further section discusses the search for fifth forces, violations of the equivalence principle and variation of fundamental constants.

Each chapter is usually completed by a collection of exercises that should help the advanced student to fill in the details of some of the arguments and derivations developed within the chapter. If appropriate, technical material is covered by appendices.

The book closes with a glossary of the most important concepts and with a detailed index of all relevant expressions and keywords.

Throughout the book, I have tried to highlight the many connections between the topics by referring to the relevant other parts of the book wherever appropriate. This sometimes makes it unavoidable that certain subjects such as neutrino physics are scattered throughout the book and keep appearing in various parts, for example, in Chap. 1 through their central role in electroweak interactions, in Chap. 4 as a prime example of freeze-out in the early Universe and in the context of leptogenesis, in Part IV as one of the main weakly coupled messengers for processes from MeV scales up to the highest energies, in Chap. 14 as possible dark matter candidates in the form of sterile neutrinos and finally in Chap. 16 as probes of Lorentz symmetry violations. Similarly, magnetohydrodynamics is first introduced in a general way in Sect. 3.6, is then applied to primordial magnetic fields in Sect. 4.6, and plays a role again in the acceleration and propagation of charged cosmic rays discussed in Chaps. 6 and 7, respectively. Dark matter is first mentioned in Chap. 2 in the context of extensions of the Standard Model and is next motivated by observations of the cosmic microwave background discussed in Sect. 4.2. Its production through freeze-out in the early Universe is presented in Sect. 4.3 whereas its role in the “late Universe” and the search for it is the subject of Chap. 14, followed by Chap. 15 on light scalar or axial scalar dark matter. While the spreading of such topics throughout the book sometimes may make the text somewhat more difficult to read, I still think it is highly preferable to present a strongly intertwined story rather than a linear narrative of disconnected subjects.

There are thus several ways of reading this monograph or parts of it or using it for lectures. The reader familiar with the Standard Model of particle physics and cosmology can use Parts I and II as a reference without the need to go through it in detail while reading some of the following chapters of his or her interest fully. Readers interested in the topics mentioned above that are more spread throughout the book could follow the order laid out above in this context. Finally, readers less familiar with the foundations of particle physics, astrophysics and cosmology could use Parts I and II as an introduction to these topics. Lecturers could combine the materials in similar ways, depending on the topic and goal of the course. In this way I hope this monograph will be useful to audiences of various backgrounds.

Corrections, errata and solutions to some of the problems will be posted at <https://www.desy.de/~sigl/astroparticle-pheno/book.html>

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Acronyms

AGN	Active Galactic Nucleus
ALP	Axion-Like Particle
BAO	Baryon Acoustic Oscillations
BBN	Big Bang Nucleosynthesis
CDM	Cold Dark Matter
CEL	Continuous Energy Loss
CL	Confidence Level
CM	Center of Mass
CMB	Cosmic Microwave Background
CR	Cosmic Ray
DPP	Double Pair Production
EBL	Extragalactic Background Light
EGMF	Extragalactic Magnetic Field
GRB	Gamma Ray Burst
ICS	Inverse Compton Scattering
ISCO	Innermost Stable Circular Orbit
LIV	Lorentz Invariance Violation
LSP	Lightest Supersymmetric Particle
MOND	MODified Newtonian Dynamics
PP	Pair Production
QCD	Quantum Chromodynamics, the gauge theory of the strong interaction
QED	Quantum Electrodynamics, the gauge theory of the electromagnetic interaction
SNR	SuperNova Remnant
SNR	Signal-to-Noise Ratio
SUSY	Supersymmetry
TPP	Triplet Pair Production
UHE	Ultra-High Energy, usually energies larger than 10^{18} eV
UHECR	Ultra-High Energy Cosmic Ray
WIMP	Weakly Interacting Massive Particle
WISP	Weakly Interacting Sub-eV Particle

Notations and Conventions

We use the $(+, -, -, -)$ sign convention for the space-time metric where the 0-component corresponds to time and the others to space, such that $X = (x^\mu) = (c_0t, x^1, x^2, x^3) = (c_0t, \mathbf{r})$. Three vectors are denoted by boldface and their three-norm by the same character without boldface, for example $r = |\mathbf{r}|$.

If not otherwise stated, we will use natural units in which the vacuum speed of light, Planck's constant and Boltzmann's constant are all unity, $c_0 = \hbar = k_B = 1$. In cgs units these constants are given by

$$c_0 = 2.99792458 \times 10^{10} \frac{\text{cm}}{\text{s}}$$

$$\hbar = 6.62607004 \times 10^{-27} \frac{\text{cm}^2 \text{g}}{\text{s}}$$

$$k_B = 1.38064852 \times 10^{-16} \frac{\text{cm}^2 \text{g}}{\text{s}^2 \text{K}}.$$

For electromagnetism in general we will use Gaussian natural units in which the electric permittivity and the magnetic permeability of the vacuum are given by $\epsilon_0 = 1/(4\pi)$ and $\mu_0 = 4\pi$, respectively. Also used often are Lorentz–Heaviside units for which $\epsilon_0 = \mu_0 = 1$.

In Gaussian natural units any dimensionful quantity can be expressed in powers of the energy unit, for which usually a giga electronvolt (GeV) is used. In gravitational physics sometimes all energies and masses are expressed in units of the Planck mass so that all quantities can be expressed in terms of dimensionless numbers. However, we will keep explicit any dependencies on the Planck mass M_{Pl} . In Table 1 we express the most important units in powers of GeV. With this, it is often convenient to perform calculations in powers of GeV and the final result can be reconverted to the appropriate units in the third column of Table 1.

Table 1 Conversion of the most frequently used units into powers of GeV

Physical quantity	Full unit name	Conversion
Energy	Joule	$1\text{ J} = 6.242 \times 10^9 \text{ GeV}$
Energy	erg	$1\text{ erg} = 6.242 \times 10^2 \text{ GeV}$
Temperature	Kelvin	$1\text{ K} = 8.622 \times 10^{-14} \text{ GeV}$
Mass	gram	$1\text{ g} = 5.601 \times 10^{23} \text{ GeV}$
Mass	solar mass	$1M_{\odot} = 1.989 \times 10^{33} \text{ g} = 1.116 \times 10^{57} \text{ GeV}$
Mass	Planck mass	$1M_{\text{Pl}} = 1.2209 \times 10^{19} \text{ GeV}$
Length	centimeters	$1\text{ cm} = 5.068 \times 10^{13} \hbar c_0 \text{ GeV}^{-1}$
Length	astronomical unit	$1\text{ AU} = 1.496 \times 10^{13} \text{ cm} = 7.581 \times 10^{26} \hbar c_0 \text{ GeV}^{-1}$
Length	mega parsecs	$1\text{ Mpc} = 3.086 \times 10^{24} \text{ cm} = 1.564 \times 10^{38} \hbar c_0 \text{ GeV}^{-1}$
Length	Planck length	$l_{\text{Pl}} = \hbar/(c_0 M_{\text{Pl}}) = 8.191 \times 10^{-20} \hbar c_0 \text{ GeV}^{-1}$ $l_{\text{Pl}} = 1.616 \times 10^{-33} \text{ cm}$
Cross section	barn	$1\text{ barn} = 10^{-24} \text{ cm}^2 = 2.568 \times 10^3 (\hbar c_0)^2 \text{ GeV}^{-2}$
Time	second	$1\text{ s} = 1.519 \times 10^{24} \hbar \text{ GeV}^{-1}$
Time	year	$1\text{ y} = 3.156 \times 10^7 \text{ s} = 4.794 \times 10^{32} \hbar \text{ GeV}^{-1}$
Time	Planck time	$t_{\text{Pl}} = \hbar/M_{\text{Pl}} = 5.391 \times 10^{-44} \text{ s}$
Angular frequency	Hertz	$1\text{ Hz}/(2\pi) = 1\text{ rad s}^{-1} = 6.582 \times 10^{-25} \text{ GeV}/\hbar$
Power	erg per second	$1\text{ erg s}^{-1} = 4.108 \times 10^{-22} \text{ GeV}^2/\hbar$
Power	Watt	$1\text{ W} = 1\text{ J s}^{-1} = 4.108 \times 10^{-15} \text{ GeV}^2/\hbar$
Elementary charge	Coulomb	$e = 0.0854(4\pi\epsilon_0)^{1/2} = 1.602 \times 10^{-19} \text{ C}$
Magnetic field	Gauss	$1\text{ G} = 6.91 \times 10^{-20} \text{ GeV}^2/(4\pi\epsilon_0)^{1/2}/(\hbar c_0)^{3/2}$

Table 2 Frequently used constants

Physical constant	Symbol and value
Electron mass	$m_e = 5.11 \times 10^{-4} \text{ GeV}/c_0^2$
Proton mass	$m_p = 0.9383 \text{ GeV}/c_0^2$
Fermi constant	$G_{\text{F}} = 1.166 \times 10^{-5} (\hbar c_0)^3 \text{ GeV}^{-2}$
$W^{\pm}$ boson mass	$m_{W^{\pm}} = 80.385 \text{ GeV}/c_0^2$
Z^0 boson mass	$m_{Z^0} = 91.188 \text{ GeV}/c_0^2$
Gravitational constant	$G_{\text{N}} = \hbar c_0/M_{\text{Pl}}^2 = 6.709 \times 10^{-39} \hbar c_0^5 \text{ GeV}^{-2}$
CMB temperature	$T_0 = 2.725\text{ K} = 2.349 \times 10^{-13} \text{ GeV}/k_{\text{B}}$
Hubble constant	$H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1} = 2.133 \times 10^{-42} h \text{ GeV}/\hbar$
Age of the Universe	$t_0 = 1.381 \times 10^{10} \text{ y}$
Critical density	$\rho_{c,0} = 3c_0^2 H_0^2/(8\pi G_{\text{N}}) = 8.098 \times 10^{-47} h^2 \text{ GeV}^4/(\hbar c_0)^3$

In Table 2 we list a few frequently used constants.

Unfortunately it was sometimes unavoidable to use the same symbol for more than one quantity. In such cases it is usually clear from the context which meaning

applies. Below we list the symbols that are most often used throughout the text with the physical quantities they stand for, including multiple meanings. They are also explained in the text where they occur for the first time. Three-vectors are generally denoted in boldface.

Latin Symbols

$a, b, \dots, h$	Early lower case Latin indices
$i, j, \dots, v$	Late lower case Latin indices, often used for spatial components when running from 1 to 3
$a(t) = R(t)/R_0$	Dimensionless cosmological scale factor for radius $R(t)$ and $R_0 = R(t_0)$
$a(t, \mathbf{r})$	Used for axial scalar axion-like fields
$a_i(\mathbf{p}), a_{\mathbf{p}}, b_i(\mathbf{p}), b_{\mathbf{p}}$	Annihilation operators for a particle or an antiparticle of flavor i , if more than one, and momentum $\mathbf{p}$
$a_i^\dagger(\mathbf{p}), a_{\mathbf{p}}^\dagger, b_i^\dagger(\mathbf{p}), b_{\mathbf{p}}^\dagger$	Creation operators for a particle or an antiparticle of flavor i , if more than one, and momentum $\mathbf{p}$
a_Z	Bohr radius of an atom with atomic number Z
c_0	Speed of light in vacuum
c_s	Speed of sound
cc	Complex Conjugate
d, d_L	Often used for distances, in particular luminosity distance d_L
e	The fundamental (positive) charge unit. In the Gaussian natural units generally used here one has $e = 0.0854$
f	Used for frequency, in particular for gravitational waves
$f(\mathbf{p}), f_i(\mathbf{p}), f(E)$	Occupation numbers as function of momentum $\mathbf{p}$ or energy E
g	Dimensionless coupling constant of the gauge group $SU(2)_L$ in electroweak theory.
g'	Dimensionless coupling constant of the hypercharge gauge group $U(1)_Y$ in electroweak theory
g_s	Dimensionless coupling constant of the color gauge group $SU(3)_c$ of QCD
$g_r(T)$	Effective number of relativistic degrees of freedom at temperature T
$g_{\mu\nu}$	A general four-metric in general relativity
h	Hubble constant H_0 in units of $100 \text{ km s Mpc}^{-1}$
h	Gravitational wave strain
h_i	Yukawa couplings
$h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$	Deviation of general four-metric in general relativity from the Lorentz metric in perturbation theory, often used for gravitational waves.
h.c.	Stands for hermitian conjugate

$\hbar = h/(2\pi)$	Planck's constant which is the natural unit of angular momentum and the action
j	Fluxes in units of particles per area and time (and per solid angle if diffuse)
j	Also used for total angular momentum number
k_B	Boltzmann's constant
l	Typically used for length scales
l	Also used for orbital angular momentum quantum number
m	Mass, typically of a test particle or a microscopic body
m_a	Axion or ALP mass
m_e, m_p, m_n, m_N	Electron, proton, neutron and nucleon mass
m_W, m_Z, m_H	W -boson, Z -boson, and Higgs boson mass
$n, n(\mathbf{r})$	Number densities as function of location $\mathbf{r}$
n	Sometimes also used for an index of refraction
p	Used for the absolute value of the momentum or for the pressure, depending on the context
p_F	Fermi momentum
p_*	Absolute value of momentum in the center of mass frame
p_{ij}	Stress tensor
q	Used for an electric charge and for the absolute value of the three-momentum transfer
r	Typically used for radial length scales
r	In inflation theory used for the tensor to scalar ratio
r_s	Schwarzschild radius
r.m.s.	Root mean square, i.e. the square root of the average of the square of a quantity
s	Spin quantum number
s, t, u	Mandelstam variables
t_α	Hermitian generators of the Lie algebra of a gauge group with gauge group index α
t, T, τ	Typically used for timescale
t_0	Age of the Universe
v	Used for absolute velocities and the vacuum expectation value of the Standard Model Higgs field
w	Used for enthalpy density and the equation of state parameter in $p = w\rho$
$x = m/T$	Temperature variable for a mass m in cosmology
y	Compton optical depth, y -parameter
z	Redshift
A	Atomic mass number or area
A_μ^α	Hermitian gauge potential and its real valued gauge components for an abelian or non-abelian gauge theory with space-time index μ and gauge group index α
$C_{\alpha\beta}^\gamma$	Structure constants for Lie algebras of gauge groups

$D_{\alpha\beta\gamma}$	Constants entering divergence of anomalous currents
$D(p), D(E)$	Momentum or energy dependent diffusion coefficient
D_μ	The covariant derivative with respect to space-time coordinate x^μ , used both for general relativity and for non-gravitational interactions
E, E_p	Energy of a single particle or field configuration of momentum $\mathbf{p}$
F	Used for the free energy
F_i	Sometimes also used for energy flux of a species i
$F(Q)$	Form factors as functions of four-momentum transfer
$F^{\mu\nu}, F_{\mu\nu}^\alpha$	Hermitian field strength tensor and its real valued gauge components for an abelian or non-abelian gauge theory with space-time indices μ, ν and gauge group index α
$\tilde{F}^{\alpha, \mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\kappa\sigma} F_{\kappa\sigma}^\alpha$	Dual field strength tensor
G_F	Fermi's constant
G_N	Newton's constant
H	The Hamiltonian or Hamilton function of a system
$H \equiv \mathbf{S} \cdot \mathbf{p}/p$	Helicity of a particle with spin $\mathbf{S}$ and momentum $\mathbf{p}$
$H = \mathcal{H}/V$	Also used for magnetic field helicity density
$H, H(T), H(z)$	Hubble rate and its temperature or redshift dependence
H_0, h	Hubble rate today, $h = H_0/(100\text{kms}^{-1}\text{Mpc}^{-1})$
$I, I(\mathbf{n})$	Used for total intensity or intensity per solid angle in direction $\mathbf{n}$ which sometimes is number flux per area and sometimes power per area
J_μ^α	Real valued gauge currents for gauge group index α
L	The Lagrangian or Lagrange function of a system
L	Also used for length scales, luminosities, lepton number or latent heat
M	Mass, typically of a macroscopic body
M_*	Effective Planck mass in D dimensions
$M_\odot$	Solar mass
M, M_{if}	Transition matrix element
M_{Pl}	Planck mass
N	Used for total numbers
N_{eff}	Effective number of relativistic neutrino flavors in cosmology
P, p	Used for probabilities
P	Also used for power
Q	Used for the four-momentum transfer or its norm $ Q^2 ^{1/2}$ as an energy scale
R	Typically used for rates, radii or the Ricci scalar in general relativity
$R(t)$	Typically used for the dimensionful scale factor in cosmology

S	Used for actions and entropy
S	Also used for the total spin quantum number
S_{if}	Transition or S-matrix element from initial to final state
SD	Spin-dependent
SI	Spin-independent
T	is also used for temperature
$T_{\mu\nu}$	Energy-momentum tensor
U	Total energy of a system
V	Used for volumes
$W(q, k)$	Transition rate from q to k
X	Used for massive dark matter particles
X	Also used for column depth, grammage
Y	Number of particles per unit of entropy
Z	Atomic charge number

Greek and Calligraphic Symbols

$\alpha, \beta, \dots$	Earlier Greek indices, in general used for gauge group components
$\kappa, \lambda, \mu, \nu, \dots$	Greek indices, often used for space-time components when running from 0 to 3
$\alpha_{\text{em}} = e^2/(4\pi\epsilon_0\hbar c_0)$	Electromagnetic fine structure constant
$\alpha_s = g_s^2/(4\pi\hbar c_0)$	Strong fine structure constant
β	Used for the plasma beta, the ratio of thermal and magnetic pressure
ξ	Used for compactness parameter $G_N M/R$ of mass M and radius R
ϵ_V	Used for one of the slow roll parameters of inflation
ε	Sometimes used for energy of a low energy background photon
$\epsilon^{\mu\nu\kappa\sigma}$	Completely antisymmetric Levi-Civita tensor with the sign convention $\epsilon^{0123} = 1$
γ	Adiabatic index
γ	Sometimes also used for rates
γ^μ	Dirac matrices
η	Often used for electromagnetic resistivity, or conformal time
η_B	Baryon number per photon in cosmology
η_V	Used for one of the slow roll parameters of inflation
$\eta_{\mu\nu}$	Lorentz metric
θ	Theta parameter in QCD
θ_0	Vacuum mixing angle

$\kappa(\theta)$	Convergence as function of image direction θ in gravitational lensing
λ	Used for affine parameter, wavelength and certain length scales
λ	Also used for dimensionless coupling constants
μ	Reduced mass
μ_B	Bohr magneton as unit for magnetic dipole moments
μ_i	Chemical potential of species i
$\mu_5 = (\mu_L - \mu_R)/2$	Chiral chemical potential
ν	Often used for kinematic viscosity
ν	In radio astronomy also used for frequency
ϕ	Scalar fields, scalar potential in electrodynamics
π_{ij}	Viscous stress tensor
ρ	Used for energy or mass densities
ρ	Also used for quantum mechanical density matrices
ρ_c	Critical energy density in cosmology
ρ_{em}	Electric charge density
$\rho_{\mathbf{p}}$	Density matrix for flavor mixed states as a function of momentum $\mathbf{p}$
σ	Often used for electromagnetic conductivity and for cross sections
σ_A	Often used for the width of the distribution of a quantity A
σ_{SB}	Stefan–Boltzmann constant
σ_T	Thomson cross section
$\langle \sigma v \rangle$	Thermal average of cross section times relative velocity
τ	Also used for optical depths
ω	Angular frequency
ω_c	Cyclotron frequency
ω_g	gyro-(Larmor) frequency
Γ	Typically used for Lorentz factors
$\Gamma^\mu_{\rho\sigma}$	Christoffel symbols, in particular for the metric-induced Levi-Civita connection
Λ	Often used for a cosmological constant or vacuum energy density
Λ_{QCD}	Energy scale of the strong interactions, confinement scale
Ω	Used for the grand canonical potential
Φ	Particle injection flux per energy, volume and time, or Newtonian potential
Φ_B	Magnetic flux
$d\Omega$	Phase space element
$\Omega_\nu, \Omega_m, \Omega_r$	In cosmology the dimensionless energy densities of the vacuum, matter and radiation, relative to the critical density
$\mathcal{C}$	Denotes contours
$\mathcal{D}$	Doppler factor
$\mathcal{F}$	Free energy density in cosmology

$\mathcal{H} \equiv \int d^3\mathbf{r} \mathbf{B} \cdot \mathbf{A}$	Global helicity of a magnetic field $\mathbf{B}$ described by a vector potential $\mathbf{A}$
$\mathcal{L}$	Used for Lagrange densities
$\mathcal{L}_V$	The Lie derivative of a tensor with respect to a vector field V^μ
$\mathcal{M}$	Mach number
$\mathcal{M}_{if}$	Reduced transition matrix element from initial to final state
$\mathcal{P}f(x)$	Principal part of an integral of a function $f(x)$ of a real variable x with a singularity
$\Re(\mathfrak{f}), \Im(\mathfrak{f})$	Real and imaginary part of a complex quantity f , respectively
$\mathcal{W}^{\mu\nu}(Q)$	Medium structure function of four-momentum transfer Q

Vectors

$\mathbf{a}$	Acceleration
$\mathbf{f}$	Force per volume
$\mathbf{k}$	Three-dimensional wave number
$\mathbf{j}$	Three-dimensional current densities
$\mathbf{j}_{\text{em}}$	Three-dimensional electromagnetic current density
$\boldsymbol{\mu}_e, \boldsymbol{\mu}_m$	Electric and magnetic dipole moments
$\mathbf{m}$	Sometimes used for magnetic dipole moment
$\mathbf{n}$	Unit normal vector
$\mathbf{p}$	Three-dimensional momentum
$\mathbf{r}$	Spatial three-vector, in particular for location
$\boldsymbol{\sigma}$	Pauli matrices
$\mathbf{v}, \beta$	Velocity three-vector
$\mathbf{v}_A$	Alfvén velocity
$\mathbf{A}$	Three-vector potential in electrodynamics
$d\mathbf{A}$	Surface area element
$\mathbf{E}, \mathbf{B}$	Electric and magnetic field strength three-vector
$\mathbf{F}$	Force
$\mathbf{G}$	Torque
$\mathbf{J}$	Total angular momentum
$\mathbf{L}$	Orbital angular momentum
$\mathbf{M}$	Used for torque per area or magnetic dipole moment
p, P, P^μ	Four-momentum
$\mathbf{S}$	Used for the spin of a particle or the Poynting vector of an electromagnetic field
$d\mathbf{S}$	Surface element
$\boldsymbol{\omega} = \nabla \times \mathbf{v}$	Vorticity of a velocity field $\mathbf{v}$
$\boldsymbol{\Omega}$	Angular frequency three-vector
x, x^μ	Space-time four-vector