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Navier–Stokes Flow Around a Rotating Obstacle

Mathematical Analysis of its Asymptotic
Behavior

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To the memory of Prof. Jindřich Nečas

Preface

The book is devoted to the mathematical analysis of the asymptotic behavior of the motion of viscous fluid around rotating and translating bodies. The work is based on the articles published during 2010–2016 which we were doing together with Paul Deuring. We would like to thank him for his wonderful collaboration and support for this project. We only regret that he could not join our project.

Š.N. would like to express her deep thanks to Prof. G.P. Galdi for introducing her to this wonderful subject. Second, she would like to thank her family—her mother Zdeňka, her sister Jindra and children—Martin, Jan and Lucie for their great support. S.K. would like to thank his wife Dagmar for her support and patience.

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