Gas Dynamics

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Gas Dynamics

An Introduction with Examples
from Astrophysics and Geophysics

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Foreword

The main objective of this book is to lay the foundations of gas and fluid dynamics. It does so by developing the basic equations from scratch, building on the (assumed) knowledge of students of Classical Mechanics. In this way, we can consider the mathematical properties of flows and discuss such things as conservation laws, perturbation analysis and waves, shocks, etc. Often we will consider *ideal fluids and gases*, where the influence of internal friction (called viscosity) can be neglected. Viscous flows will be discussed when we consider flows around obstacles and shocks.

Many of the examples used to illustrate various processes come from astrophysics, but now and then, I will look at geophysical phenomena.

This subject is a challenging one: In order to fully understand the intricacies of fluid mechanics/gas dynamics, one needs to know about scalars, vectors and simple (rank 2) tensors, and the vector/tensor analysis that comes with it. I can only hope that this book may serve as a readable introduction to a rich subject.

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Abraham Achterberg

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