Atlantis Studies in Mathematics for Engineering and Science

Volume 13

Series editor

Charles K. Chui, Department of Statistics, Stanford University, Stanford,
California, USA

Aims and scope of the series

The series ‘Atlantis Studies in Mathematics for Engineering and Science’ (AMES) publishes high quality monographs in applied mathematics, computational mathematics, and statistics that have the potential to make a significant impact on the advancement of engineering and science on the one hand, and economics and commerce on the other. We welcome submission of book proposals and manuscripts from mathematical scientists worldwide who share our vision of mathematics as the engine of progress in the disciplines mentioned above.

For more information on this series and our other book series, please visit our website at: www.atlantis-press.com/publications/books

Atlantis Press
8, square des Bouleaux
75019 Paris, France

More information about this series at <http://www.springer.com/series/10071>

Radomir S. Stanković · Paul L. Butzer
Ferenc Schipp · William R. Wade
Authors/Co-Authors

Dyadic Walsh Analysis from 1924 Onwards Walsh-Gibbs-Butzer Dyadic Differentiation in Science Volume 2 Extensions and Generalizations

A Monograph Based on Articles
of the Founding Authors, Reproduced in Full

in Collaboration with the Co-Authors

Weiyi Su

Yasushi Endow

Sándor Fridli

Boris I. Golubov

Franz Pichler

Kees (C.W.) Onneweer

Authors

Radomir S. Stanković
Faculty of Electronic Engineering
Nis
Serbia

Paul L. Butzer
Lehrstuhl A für Mathematik
RWTH Aachen
Aachen
Germany

Ferenc Schipp
Numerical Analysis
Eötvös Loránd University
Budapest
Hungary

William R. Wade
Mathematics and Computer Science
BIOLA University
La Miranda, California
USA

Co-Authors

Weiyl Su
Mathematics
Nanjing University
Nanjing
China

Yasushi Endow
Chuo University
Tokyo
Japan

Sándor Fridli
Numerical Analysis
Eötvös Loránd University
Budapest
Hungary

Boris I. Golubov
Chair of Higher Mathematics
Moscow Institute of Physics
and Technology (State University)
Dolgoprudny, Moscow Region
Russia

Franz Pichler
System Theory
Johannes Kepler University, Linz
Linz
Austria

Kees (C.W.) Onneweer
Department of Mathematics and Statistics
University of New Mexico
Albuquerque
USA

ISSN 1875-7642

ISSN 2467-9631 (electronic)

Atlantis Studies in Mathematics for Engineering and Science

ISBN 978-94-6239-162-8

ISBN 978-94-6239-163-5 (eBook)

DOI 10.2991/978-94-6239-163-5

Library of Congress Control Number: 2015955598

© Atlantis Press and the author(s) 2015

This book, or any parts thereof, may not be reproduced for commercial purposes in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system known or to be invented, without prior permission from the Publisher.

Printed on acid-free paper

*To the memory of Joseph Leonard Walsh and
James Edmund Gibbs*

Preface

In sixties, many algorithms for processing of single and two-dimensional signals were theoretically developed. For most of them, being based on Fourier analysis, the practical applications were rather limited by the lack of the fast computing algorithms for the (complex-valued) Fourier coefficients. Due to the pressure of concrete applications, the importance of which was related also to the political situation in the World, and especially the Cold War, theory was enriched in 1965 with the Cooley-Tukey Fast Fourier Transform (FFT), which is an algorithm for efficient, in terms of space and time, computation of the Discrete Fourier Transform (DFT) ¹. Still, in spite of the existence of FFT, the computing power of hardware at the time, was inadequate to completely support related applications. In particular, difference in the price of the operations of addition and multiplication required to compute the complex-valued Fourier spectral coefficients, motivated reformulation of the Cooley-Tukey FFT and raised a variety of FFT algorithms, as for instance, the Good-Thomas FFT, the Winograd FFT algorithm, the Rader-Brenner FFT algorithm, whose numerical instability was improved in the split radix Cooley-Tukey FFT, prime factor FFT, etc. An alternative was to look for a fast to compute transform having the good properties of the Fourier transform on the real line. This motivated the interest in Walsh (Fourier) analysis, since the Walsh functions take just two values, 1 and -1 , and, therefore, computing of the Walsh (Fourier) transform coefficients requires no multiplications. The interest was extended to the computationally even faster Haar transform based on three-valued $(1, 0, -1)$ Haar functions.

In 1969, Henning F. Harmuth, who set the foundations for the applications of Walsh functions in communication and electromagnetic theory, wrote ²

Communication theory was founded on the system of sine-cosine functions. A more general theory has become known more recently; it replaces the sine-cosine by other systems of orthogonal functions, and the concept of frequency by that of sequency. Of these systems, the Walsh functions are of great practical interest since

¹ Cooley, J.W., Tukey, J.W., "An algorithm for the machine calculation of complex Fourier series", *Math. Computation*, Vol. 19, 297-301, 1965.

² Harmuth, H.F., "Applications of Walsh functions in communications", *IEEE Spectrum*, No. 11, 1969, 82-91.

they lead to equipment that is easily implemented by semiconductor technology.

In 1972, Harmuth and his associates stated ³

Coils, capacitors, and optical lenses favor sine-cosine functions. The digital computer has other favorites, such as Walsh, Hadamard, and Haar functions, which have become increasingly significant in communications since the era of semiconductors.

Therefore, the new theory, called the discrete Walsh dyadic analysis, emerged to meet characteristics of the technology at the time with the discrete Walsh functions as the basic concept in the theory, due to their simplicity and compatibility with bistable devices used to realize the corresponding systems. Within this theory, the notion of the derivative, as an operator allowing to estimate the rate of change as well as the direction of the change of a signal, was necessary, especially regarding the applications intended. In such circumstances, due to the interest in Walsh functions and the dyadic analysis, the finite dyadic derivative, initially called the logic derivative, now called the Gibbs derivative, having Walsh functions as its eigenfunctions, was defined in 1967 by James Edmund Gibbs. ⁴

In the forty five years since the concept of Gibbs derivative was first introduced in the restricted context of a finite dyadic group ⁵, the scope of the definition has been greatly extended, many results have been obtained, and areas of application have been established ⁶. In overall, a beautiful mathematical theory of Gibbs dyadic differentiation, with the Butzer-Wagner dyadic derivative as the most prominent concept from the mathematical analysis point of view, was created and is still continuously developing.

The very high interest in Walsh and dyadic analysis in seventieths, was obvious from the organization of international workshops and conferences devoted exclusively to the applications of Walsh functions from 1970 to 1974 sponsored by the Naval Research Laboratory, and held in Washington DC, USA. Moreover, there were even two conferences on that subject in 1973, the other was the Symposium on Theory and Applications of Walsh and Other Non-sinusoidal Functions, Hatfield,

³ Harmuth, H.F., Anrews, H.C., Shibata, K., "Two-dimensional sequency filters", *IEEE Trans. Communications*, Vol. COM-20, No. 3, 1972, 312-331.

⁴ Gibbs, J.E., "Walsh spectrometry a form of spectral analysis well suited to binary digital computation", *NPL DES Repts.*, National Physical Lab., Teddington, Middlesex, England, 1967.

⁵ Gibbs, J.E., Sine waves and Walsh waves in physics, Proc. Sympos. Applic. Walsh Functions, Washington, D.C., 1970 Mar. 31-Apr. 3, 260-274, (1970).

Gibbs, J.E., "Functions that are solutions of a logical differential equation", NPL DES Rept, No. 4, iv+21 pp. (1970).

Gibbs, J.E., Millard, Margaret J., "Walsh functions as solutions of a logical differential equation", NPL DES Rept, No. 1, 9 pp. (1969).

⁶ Butzer, P.L., Wagner, H.J., "Approximation by Walsh polynomials and the concept of a derivative", Proc. Sympos. Applic. Walsh Functions, Washington, D.C., 388-392, (1972).

Butzer, P.L., Wagner, H.J., "On a Gibbs-type derivative in Walsh-Fourier analysis with applications", Proc. Nat. Electron. Conf., 27, 393-398, (1972).

UK. At that time, there were expectations that the dyadic calculus and the derivative might have a role in information sciences and computing comparable to that of the Newton-Leibniz derivative in classical mathematical analysis and mechanics.

This interest in Walsh (Fourier) analysis was gradually reduced in the next decades for various reasons, partially due also to the advent of computing power overcoming the restrictions to use complex computations in classical Fourier analysis, and also for the advent of alternative theoretical tools. In these settings, the Walsh and dyadic analysis become a standard classical tool in certain areas, with a well established role and place. The related spectral methods are a standard subject within the scope of several international conferences and journals. It is, however, a feeling that this area has a great potential and is still waiting for wider and more important applications.

The situation in present computing technologies reminds that in former times in the sense that a revolutionary future development is necessary. Further increasing the number of transistors and elementary gates does not provide right solution, since the present technology is approaching to its physical limits in that respect. Improving the computational power by increasing the clock frequency is also limited by high energy dissipation and heating, both features hardly acceptable in practice. Thus, some essentially new computing technologies are needed and nothing like that still appears at the horizon, at least looking from the engineering perspective. Alternative approaches such as, for instance, quantum computing, DNA computing, and similar, are encouraging, but still far from immediate concrete engineering applications.

Multi-core central processing units (CPU) and Graphic processing units (GPU) that are recently open for general purpose computing (GPGPU) provide a short break in the search for new technologies. In these devices, the situation is reverse to that in seventies, since now computing is cheap, and memory latency and memory bandwidth are the critical factors. Therefore, the same rethinking of algorithms as that in devising various FFT algorithms in seventies is required, just in the opposite sense. Instead of reducing the number of operations, we should minimize the amount of memory transfers.

These examples discussed above, illustrate that taking a fresh look into existing theories, as dyadic calculus in general and the dyadic differentiation as a facet of it in particular, may help in borrowing old or devising new ideas that might appear useful presently in various aspects. This is an excuse which we would be glad to offer and will be happy if accepted for bringing to public notice the former work in this area. We are not so enthusiastic to think that dyadic differential calculus will be immediately useful in revolutionizing computing technologies. It however might raise new questions and challenging problems. Most of them would be probably trivial to solve, some more difficult as student exercises, but a couple of them could be really interesting.

Acknowledgments

The four editors of this monograph, in particular Radomir S. Stanković and Paul L. Butzer, express their thanks for its preparation to the authors of the monograph together with their former students. The initiator of dyadic differentiation, James Edmund Gibbs, National Physical Laboratory, Teddington, UK, died much too early, in January 2007, and so could, most unfortunately, not participate in our present project.

Mrs. Merion Gibbs reported that according to her husbands diary, his first definition of his "logical derivative" was written in January 1967. Of his 27 publications on dyadic derivatives six were written together with Dr. Brian Ireland, Bath University, UK, who joined Dr. Gibbs in the work on this subject in 1971.

The joint work of authors towards the present monograph, which took place over a period of almost four years, was very constructive and unbelievably harmonious, and even during the time from 1969 until 1990 when the chief results of dyadic Walsh analysis were developed by them. This is by no means self-evident since the authors come from ten countries, namely Austria, Canada, China, England, Germany, Hungary, Japan, Russia, Serbia, USA, with quite different mathematical/scientific traditions.

The majority of these, together with some of their former students, only met for the first time at the Workshop "Theory and Applications of Gibbs Derivatives", conducted by Radomir S. Stanković at Kupari- Dubrovnik, Yugoslavia, on September 26-28, 1989.

All authors express their special thanks to Charles Chui, Stanford University, for accepting our monograph as Editor of the series "Atlantis Studies in Mathematics for Engineering and Science" of Atlantis Press, Paris, without any hesitations. It is quite common for selected or complete publications of one author (which could cover several fields) to appear in book-form. But this is by no means so for selected papers of all founding authors of one field, even together with reader-friendly reviews of their own papers, to appear in book-form. Further, the authors thank Dr. Keith Jones for handling all matters concerned with Atlantis Press, Paris, in an efficient and friendly manner, as well as Springer for production, marketing, and distribution of this book. In particular, thanks are due to Ms. Devi Ignasy, the Project Coordinator, and Ms. Gajalakshmi Sundaram, the Production Editor, from Springer, for excellent processing of the manuscript, especially for very good care of reprinted papers, the source versions of some of them which were not so clear and easily readable.

The Editors

Organization of the Book

This second volume is intended to discuss various generalizations of the dyadic Gibbs derivatives. The considerations start by Chapter 1 presenting a concise, but very informative presentation of the work in the area by scholars in China written by Weiyi Su who is a leading expert in the subject. The discussions of research in China continues with three papers that were initially published in Chinese, and in this book are for the first time presented in English, due to the translation by Prof. Weiyi Su and her associates. We also reprinted a paper by Prof. He Zelin discussing fractional dyadic derivatives.

This part of the book ends with the Chapter 2 discussing the former and recent work on the generalization of dyadic differential operators into pseudo-differential operators on local fields.

Chapter 3 provides the insight into the work in the area of dyadic differentiation in former Soviet Union and serves as an introduction into the presentation on more recent results of the research in the subject area in Russia discussed in Chapter 4. Different aspects of generalizations of dyadic Gibbs differentiation on Vilenkin groups are discussed in Chapters 5 and 6, while Chapter 7 discusses extensions of the concept to arbitrary not necessarily Abelian groups. As the former work providing foundations, we reprinted a report on the subject by Gibbs and J. Simpson in 1974. The chapter 8 presents a short review of situation in which R.S. Stanković joined the work in dyadic analysis and in particular the Gibbs and Butzer-Wagner differentiation and related topics as an illustration of the work in this area in Serbia at that time.

For practical applications, primarily these engineering oriented, it is important to provide fast computing methods for the dyadic Gibbs derivatives and their generalizations. With this motivation, Chapter 9 discusses efficient computing of Gibbs derivatives on finite Abelian groups and provides a comparison of efficiency of these methods on general purpose computers with computations on Central Processing Units (CPU) and on Graphics Processing Units (GPUs). Two reprinted papers presented at the First International Workshop on Gibbs Derivatives held in 1989, highlight certain aspects of the related theory by discussing, respectively, the relationships between the classical Newton-Leibniz derivative and the Gibbs derivatives, and extensions to finite non-Abelian groups.

Applications of Gibbs derivatives to stochastic processes, with a particular emphasis to stochastic processes called Walsh harmonizable processes is considered in Chapter 10. A reprinted related paper published in 1984 provides a necessary background.

The chapter 11 presents the involvement of Y. Endo in the research work in dyadic analysis with application to dyadic stationary processes. This chapter illustrates the research in the area in Japan at that time.

This volume ends with Chapter 12 presenting open problems in theory and applications of dyadic Gibbs derivatives and their generalizations.

Contents

1	Early Work in Gibbs Differentiation in China	1
	Weiyl Su	
1.1	Research in Butzer-Gibbs Calculus in China	
	- A glance of p -type calculus	2
	References	3
1.2	The Generalized Logical Derivative and Its Applications	5
1.2.1	Introduction	5
1.2.2	The p -adic Logical Derivatives	7
1.2.3	Bernstein Type Inequality and Bernstein Type Theorem	11
	References	14
1.3	The Generalized Walsh Transform and An Extremum Problem	15
1.3.1	Introduction	15
1.3.2	Properties of W_N -type functions	16
1.3.3	The Descent Order of W_N -type Functions	18
1.3.4	p -adic Logic Derivatives	23
1.3.5	Logic derivative	24
1.3.6	Adjoint logic derivative	25
1.3.7	A Class of Extremum Problems	27
	References	32
1.4	The Logical Derivatives and Integrals-Zheng Weixing, Su Weiyl	33
1.4.1	Introduction	33
1.4.2	Notation and important results	34
1.4.3	p -adic derivatives and integrals of functions in L_G^q , $1 \leq q \leq 2$	36
	References	48
1.5	The Logical Derivatives and Integrals Second Part	49

2	Gibbs-Butzer Calculus and Pseudo-differential Operators on Local Fields	65
	Weiyl Su, Qiu Hua	
2.1	Introduction	65
2.2	Notations	66
2.3	G-B type derivative and G-B type integrals	67
2.4	Function Spaces on Local Fields	75
2.5	A Comparison of Newton calculus with G-B calculus	79
2.6	Applications of Gibbs-Butzer-type Calculus	82
2.6.1	Application to Approximation Theory on Local Fields	82
2.6.2	Application to determine function space for which a function belongs to it	83
2.6.3	Application to partial differential equations with fractal boundaries	84
2.6.4	Application to medical science	89
	References	90
3	Early Work in Dyadic Differentiation in Soviet Union - SSSR	93
	Radomir S. Stanković, Paul L. Butzer	
3.1	Interest in Dyadic Differentiation in Soviet Union	93
4	Research in Dyadic Differentiation in Russia	125
	Boris I. Golubov, Sergei S. Volosivets	
	References	128
5	Generalized Derivatives and Integrals on Vilenkin Groups	131
	Boris I. Golubov, Sergei S. Volosivets	
5.1	A. The case of compact groups	131
5.2	B. The case of locally compact groups	137
	References	141
6	Calculus on Walsh and Vilenkin Groups	145
	György Gát, Rodolfo Toledo	
6.1	Introduction	145
6.2	Results related to Vilenkin systems	148
6.3	Results related to representative product systems	151
6.4	Results related to Walsh-Kaczmarz systems	152
6.5	Results related to two-dimensional Walsh-Paley systems	153
	References	155
7	Gibbs Derivatives on Groups	157
	Radomir S. Stanković	
7.1	Introduction	157
7.2	Basic Concepts	159
7.3	Gibbs Derivatives on Finite Non-Abelian Groups	161
7.4	Summary and Recent Work	165
	References	166

8	My Research in Gibbs Derivatives on Finite Groups	169
	Radomir S. Stanković	
	References	171
9	Efficient Computation of Gibbs Derivatives on Finite Abelian Groups	211
	Radomir S. Stanković, Dušan Gajić	
9.1	Introduction	211
9.2	Definitions of Gibbs Derivatives on Finite Abelian Groups	212
9.3	Generalizations	214
9.4	Fast Fourier Transform on Finite Abelian Groups	215
9.5	Gibbs Derivatives on Finite Abelian Groups	217
9.6	Computing the Gibbs Derivatives on Finite Abelian Groups	219
9.6.1	Convolution-like algorithm	219
9.6.2	Computing in terms of partial Gibbs derivatives	219
9.7	GPGPU Computing Platform	220
9.8	Mapping the Algorithms to GPGPU Architecture	222
9.9	Experimental Results	223
	References	227
10	Gibbs Derivative and Walsh Harmonizable DSP	277
	Yasushi Endow	
10.1	Introduction	277
10.2	Q-loop and dyadic time	277
10.3	Gibbs derivative and Walsh functions	279
10.3.1	Walsh functions	279
10.3.2	Gibbs derivative	280
10.4	Dyadic system and dyadic flow	281
10.5	Dyadic stationary process	282
10.5.1	DSP in strict sense	282
10.5.2	DSP in wide sense and its W-harmonizability	283
10.5.3	Walsh harmonizability of DSP	284
10.5.4	DSP and LDP	285
10.5.5	Walsh Series and Approximation of DSP	286
10.6	Gibbs Differentiation of DSP	286
10.6.1	Gibbs Differentiability Conditions	287
10.6.2	A Linear Gibbs Differential Equation	287
10.7	Extensions and Comments	288
10.7.1	Chrestenson functions	289
10.7.2	A p -adic SP and C-harmonizability	292
10.7.3	W-Harmonizable and C-Harmonizable DSPs	293
10.7.4	Open Problems	293
	References	294

11 My Involvement in Gibbs Derivatives and Walsh Harmonizable Processes	297
Yasushi Endow	
11.0.1 Introduction	297
11.0.2 Walsh Harmonizable DSP	298
11.0.3 DSP and LDP	298
11.0.4 Walsh Series and Approximation of DSP	299
11.0.5 Gibbs Differentiability of DSP	300
11.0.6 A Linear Gibbs Differential Equation	301
11.0.7 Extensions and Comments	302
References	302
12 Open Problems in Theory and Applications of Dyadic Derivatives	325
References	326
References	327
References	328
12.1 Hardy Spaces	329
12.2 The one-dimensional dyadic derivative	330
References	332
List of Publications on Dyadic Differentiation by the Authors of the Book	333
Index	347
Biographies of Authors	351

List of Contributors

The names of the contributors to this book are given in alphabetical order.

Paul L. Butzer

Lahrstuhl A für Mathematik, RWTH Aachen, Germany
e-mail: butzer@rwth-aachen.de

Yasushi Endow

Department of Industrial and Systems Engineering, Chuo University, Tokyo,
112-8551 Japan, e-mail: endow@indsys.chuo-u.ac.jp

Sándor Fridli

Department of Numerical Analysis, Eötvös L. University, Budapest,
Pázmány P. sétány 1\C, H-1117 Hungary, e-mail: fridli@numanal.inf.elte.hu

Dušan Gajić

Dept. of Computer Science, Faculty of Electrical Engineering, Niš, Serbia,
e-mail: dule.gajic@gmail.com

György Gát

College of Nyíregyháza, Inst. of Math. and Comp. Sci., Nyíregyháza,
P.O.Box 166., H-4400, Hungary, e-mail: gatgy@nyf.hu

Ushangi Goginava

Dept. of Mathematics, Faculty of Exact and Natural Science, Tbilisi State
University, Tbilisi, Georgia, e-mail: ushangi.goginava@tsu.ge

Boris I. Golubov

Chair of Higher Mathematics, Moscow Institute of Physics and Technology
(State University), 141700 Dolgoprudny, Moscow Region, Russia,
e-mail: golubov@mail.mipt.ru

Qiu Hua

Mathematics Department, Nanjing University, China, e-mail: qiuhua@nju.edu.cn

Kees (C.W.) Onneweer

Department of Mathematics and Statistics, University of New Mexico,
Albuquerque, USA, e-mail: onneweer@math.unm.edu

Franz Pichler

Johannes Kepler University, Linz, Austria, e-mail: Franz.Pichler@jku.at

Ferenc Schipp

Department of Numerical Analysis, Eötvös L. University, Budapest,
Pázmány P. sétány 1/C, H-1117 Hungary, e-mail: schipp@numanal.inf.elte.hu

Radomir S. Stanković

Dept. of Computer Science, Faculty of Electronic Engineering, University of Niš,
Niš, Serbia, e-mail: Radomir.Stankovic@gmail.com

Weiyi Su

Mathematics Department, Nanjing University, China, e-mail: suqiu@nju.edu.cn

Rodolfo Toledo

College of Nyíregyháza, Inst. of Math. and Comp. Sci., Nyíregyháza,
P.O.Box 166., H-4400, Hungary, e-mail: toledo@nyf.hu

Sergey S. Volosivets

Faculty of Mechanics and Mathematics, Saratov State University,
410012 Saratov, Russia, e-mail: volosivetsSS@mail.ru

William R. Wade

Mathematics and Computer Science, Biola University, La Mirada CA 90639, USA
e-mail: william.wade@biola.edu

Hans Josef Wagner

Krauthausener Str. 6A, 52223 Stolberg-Dorff, Germany

Ferenc Weisz

Department of Numerical Analysis, Eötvös L. University
H-1117 Budapest, Pázmány P. sétány 1/C., Hungary, e-mail: weisz@inf.elte.hu