Atlantis Studies in Probability and Statistics

Volume 6

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AMSTERDAM—PARIS—BEIJING

ATLANTIS PRESS

Atlantis Press

29, avenue Laumière

75019 Paris, France

More information about this series at <http://www.atlantis-press.com>

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Records via Probability Theory

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ISSN 1879-6893 ISSN 1879-6907 (electronic)
Atlantis Studies in Probability and Statistics
ISBN 978-94-6239-135-2 ISBN 978-94-6239-136-9 (eBook)
DOI 10.2991/978-94-6239-136-9

Library of Congress Control Number: 2015942749

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To my parents and my wife, Masuda
Mohammad Ahsanullah

To my wife, Ludmila
Valery B. Nevzorov

Preface

“Record, Record, Record!”, “The oldest Olympic Record was beaten!”, “New World Record result in high-jumping!”. Every day, we wait for such kind of headlines in newspapers, on the Internet, in TV announcements.

We hurry to call our friends and tell them about a new record value of the summer temperature in California, which was fixed yesterday, and about the highest level of today’s earthquake somewhere in India.

We collect all editions of the Guinness Book of Records. Not only the Guinness Book but all tables of record results in various domains of human activities which are very popular among citizens all over the world.

Indeed, we regard any record result of known or unknown persons as the progress of all humankind (and, in particular, as our own progress).

It fell out that the record topic has become very popular among specialists in probability and mathematical statistics. A lot of papers (beginning with Chandler (1952)), in which various aspects of the mathematical theory of records were considered, have appeared during the past 60 years. Some monographs have been published on the records topic (see, for example, Ahsanullah (1995), Ahsanullah and Nevzorov (2001a), Arnold, Balakrishnan and Nagaraja (1998), Nevzorov (2000, 2001)).

Our readers can ask: “What is the reason for publishing a new book on records when there are so many monographs on this topic?”

The matter is that each coming year gives many new results on record times and record values. Some new record schemes have been suggested by various authors. A number of new methods for studying records have been presented in new publications. That is why we offer to our readers a book, which includes, in particular, some new materials about records. This monograph will help you to clear the situation with the modern theory of record values. You will find a lot of examples and exercises here, which will enable you to get a good experience of becoming familiar with the theoretical part of this theory.

The theory of records is connected very closely with the theory of order statistics. In some sense our book can be considered as the continuation of the Ahsanullah, Nevzorov, and Shakil (2013) edition, which provides an introduction

to the world of order statistics. We suggest a lot of exercises, for which solutions are given at the end of the corresponding chapters. These exercises will help our readers to determine their level of comprehension of the material presented. Indeed, in order to understand the definitions and results given in the book, it is not necessary to read preliminarily any other book on order statistics. Some useful material from the theory of order statistics is presented in the first part of our book.

You are welcome to open the world of records for yourself!

With great pleasure we recall our friends N. Balakrishnan, P. Deheuvels, H.N. Nagaraja to whom we express our deep gratitude. The exchange of ideas and the joint work with our above-mentioned colleagues are of great importance to our scientific and publishing activities. We are also grateful to our family members for their constant encouragement and support.

We wish to express our gratitude to Prof. Chris Tsokos for valuable suggestions about the manuscript. The first author thanks Z. Karseen and K. Jones for interesting discussions at a meeting in Athens, Greece, for the publication of this book.

The work of the first author was partially supported by a summer research grant and sabbatical leave from Rider University. The work of the second author was partially supported by Saint Petersburg State University grant N 6.38.672.2013.

Mohammad Ahsanullah
Valery B. Nevzorov

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