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Principles of Mathematical Economics

ATLANTIS PRESS

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Editorial

Recent years have witnessed an extraordinarily rapid advance in the direction of information technology within the scientific, engineering, and other disciplines, in which mathematics play a crucial role. To meet such urgent demands, effective mathematical models as well as innovative mathematical theory, methods, and algorithms have to be developed for data information manipulation, understanding, visualization, communication, and other applications.

The Atlantis book series, "*Mathematics Textbooks for Science and Engineering* (MTSE)", is founded to meet the need of textbooks on the fundamental and core of such mathematical development, as well as their applications, that can be used for classroom teaching at the college level and beyond. For the benefit of students and readers from the interdisciplinary areas of mathematics, economics, computer science, physical and biological sciences, and various engineering specialties, contributing authors are requested to keep in mind that the writings for the MTSE book series should be elementary and relatively easy to read, with sufficient examples and exercises. We welcome submission of such book manuscripts from all who agree with us on this point of view.

This third volume, authored by Prof. Shapoor Vali, on the principles of mathematical economics, with emphasis on non-linear mathematical models, is a very valuable contribution to the MTSE series. Written for students in economics, business, management, and related fields, this textbook is self sufficient and self contained for the reader with only a basic knowledge of pre-college algebra as well as introductory micro- and macro-economics. A distinct and important feature of this nicely written textbook is the connection of the mathematical models, developed and formulated in each chapter, to specific real-world problems. We welcome this textbook to the MTSE book series.

Menlo Park, CA — Charles K. Chui

To Firoozeh, Behrang, and Seena

Preface

This book has evolved from over many years of teaching Mathematical Economics (Math-Econ, for short) and other quantitative courses at Fordham University. In my Math-Econ classes, I have used different books as text or reference. However, I always felt compelled to complement the texts by a large volume of my own notes and handouts covering areas and topics that I considered important but either not covered or not fully developed in the existing text books. So in 2009, I decided to collect my notes and develop them into an organized text book. The result was a manuscript which I have used as a text in my Math-Econ courses for the last 4 years and annually revised it based on students' feedbacks.

This textbook is written for students in economics, business, management, and related fields, for both undergraduate and introductory level graduate courses. Chaps. 1 through 7 and parts of Chaps. 8 and 9 could be covered in a one-semester undergraduate course. The entire book can be covered in a one-semester graduate course, with emphasis on Chaps. 6 through 13.

Mathematical Economics is generally a required course for students majoring in economics. It is also a widely recommended course for business schools students. This book is designed to be useful in both areas.

Economics is a quantitatively oriented discipline where mathematics plays a fundamental role in all of its related fields. Working with and manipulating numbers is an indispensable and inescapable part of economics and business studies, even in the field of economic history. A 1993 Nobel Prize in Economics was awarded to Robert Fogel of University of Chicago and Douglass North of Washington University, both economic historians, for their contribution to the application of quantitative methods to the field of economic history. Fogel and North advanced "Cliometrics",[1] a new quantitative technique for analyzing historical data and solving historical puzzles.

Lack of proficiency in mathematics is a great obstacle to learning economics. Lack of proficiency should not be interpreted only as lack of skill in manipulating algebraic or arithmetic expressions, but rather a lack of power of abstraction and

[1] Combination of 'Clio', muse and daughter of Zeus in Greek mythology, and 'metrics'. There is a Cliometric society which publishes *Cliometrica—Journal of Historical Economics and Econometric History*.

generalization. In economics, and in many other fields for that matter, power of abstraction is much more important than mechanical skills.

Mathematical Economics is not a distinct area in economics, like *International Economics*, *Money and Banking*, or *Urban and Regional Economics*. It is simply an application of mathematical tools in economics, all areas of economics, in the process of formulation and test of theories (or their purely logical aspects called *models*), and stating and providing solutions to economic "problems."

I consider Math-Econ as not simply a specific area of applied mathematics. I am sure many economists will agree with me that the main purpose of Math-Econ is not to learn more mathematics, but rather use mathematics as an aid for learning more economics. This principle has been the guiding star in writing this book. Naturally, the text contains a lot of *economics*.

A glance at various topics covered in different chapters shows that some of these topics are classic Math-Econ repertoire. But in addition to covering traditional topics, I used the same mathematical tools for formulating and solving some important Economics models that are either completely ignored or not fully explored by other textbooks, in some cases for the simple reason that they lead to nonlinearity. Almost all of mathematical text books published in the last 74 years, following R. G. D. Allen's classic *Mathematical Analysis for Economists* published in 1938, have avoided nonlinearity by framing the models such that solutions do not go beyond solving quadratic equations.

A very simple example arises in the context of standard market equilibrium model. Here we have a supply function, a demand function, and an equilibrium condition. As long as the supply and demand functions are linear or quadratic, we have no problem determining the equilibrium price and quantity, but if the supply and demand functions are nonlinear we may run into computational problem. Consider the following simple model

$$Q_d = \frac{100P + 1500}{(P^2 + 10)}$$
$$Q_s = 10P^{0.7}$$

Here demand is a rational function and supply is a polynomial of degree 0.7.

To find the equilibrium price algebraically, we follow the familiar routine

$$Q_d = Q_s \rightarrow \frac{100P + 1500}{(P^2 + 10)} = 10P^{0.7}$$

leading to

$$P^{2.7} + 10P^{0.7} - 10P - 150 = 0$$

This equation is not a run of the mill familiar equation, like linear or quadratic, and does not yield an analytical solution.

Another typical problem arises in the context of a firm production decision. Given a firm's total cost function (*TC*) and the demand function for its product, we typically want to determine the firm's profit maximizing output/price combination, its break-even level(s) of output/price and the normal profit level of output and price. We can safely do all three if the total cost function is linear or quadratic and the demand function is linear. But as soon as we introduce a more realistic cubic total cost function, a function that produces all the nice average total, average variable, and marginal cost curves, which are used in all the existing economics textbooks, then the determination of break-even and normal profit output/price leads to solving cubic equations. If we assume

$$TC = aQ^3 + bQ^2 + cQ + d$$

$$P = e - fQ$$

then for break-even(s) we need to solve $TC - TR = 0$ (where TR is the total revenue), that is the following cubic equation

$$aQ^3 + (b+f)Q^2 + (c - e)Q + d = 0$$

And for the long run normal profit scenario, not only we need $TC - TR = 0$, but we also need $MC - MR = 0$ (where MC and MR are the marginal cost and marginal revenue). This leads to solving the following cubic equation

$$2aQ^3 + (b+f)Q^2 - d = 0$$

Determining a firm's normal profit output/price when the market is competitive leads to a similar situation. In this case, we must solve

$$\frac{dATC}{dQ} = 2aQ^3 + bQ^2 - d = 0$$

So far we simply assumed that a noncompetitive firm faces a linear demand function in the market. What if the demand function is not linear? What if it is a quadratic function? A semi-log function? Or a log-linear function (a model so widely used in practice for estimating demand functions due to its desirable property of having constant price and nonprice elasticity)? In the log-linear case,

$$\ln P = \ln d - \beta \ln Q$$

a simple profit optimization routine leads to solving an equation of the form

$$3aQ^{(2+\beta)} + 2bQ^{(1+\beta)} + cQ^{\beta} - (1 - \beta)d = 0$$

if the total cost function is cubic, or

$$2aQ^{(1+\beta)} + bQ^{\beta} - (1 - \beta)d = 0$$

if the cost function is quadratic.

Modeling and solving numerous real world problems invariably leads to a nonlinear scenario, both in economics and in business. I believe in the validity of the view expressed by Naseem Talib, which is quoted in the opening page of Chap. 7 “Nonlinear Models.” The last sentence of the quote reads:

> Yesterday afternoon I tried to take a fresh look around me to catalog what I could see during my day that was linear. I could not find anything.

The introduction of nonlinearity may also lead to interestingly unexpected results; an issue ignored in current textbooks and economics literature.

We have reached a level of computational sophistication that many of these nonlinear models can be handled fairly easily. The currently available tools, some like WolframAlpha, **R**, and SAGE freely available online, or Microsoft Mathematics that can be downloaded, remove the barrier to the introduction of this type of models in economics and Math-Econ textbooks and free us from exclusively graphic presentation of many important economic models. Even in the absence of access to Internet-based mathematical software, an inherently nonlinear system can be transformed into a linear or quadratic model and subsequently solved, in many cases with small or tolerable errors. In an appendix to Chap. 6, some of these tools are introduced and in Chap. 7, and the subsequent chapters, a large number of nonlinear models are presented and solved.

My own experience is that students prefer the use of computer-based learning to paper/pencil. They only use paper/pencil when we insist that they must show their work on their weekly homework assignments. They come to class with laptops, tablets, and smart phones which are all capable of accessing the Internet and thus the full scope power of the web.

Based on my observations, it takes students not more than 30 min to learn how WolframAlpha works, along with the basics of graphing and solving equations.[2] In my undergraduate Math-Econ classes, I cover Chaps. 1 through 7 plus logarithms and exponential functions (Chap. 9) along with some sections from Chap. 8 (Additional Topics in Perfect and Imperfect Competition). After a short review of optimization models in Chap. 6, I cover the remaining chapters in a graduate Math-Econ course. I introduced both classes to WolframAlpha and Microsoft Mathematics, which both sets of students easily learn. In my graduate class, I introduced some of the tools of **R**, especially for matrix operations and applications of matrices covered in Chap. 13, along with basics of Maple. Nowadays, almost all 4-year universities with a Mathematics department have license for some mathematical software including Maple. I share with some of my grad students, who are eager to learn Maple, a series of Maple snippets (a short collection of Maple commands for solving specific problems). I have included a sample of Maple snippets at the end of Chap. 8.

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[2] Besides computational and graphic power, WolframAlpha provides students with up to date economics and business information and data.

All of these tools will undoubtedly enhance the text. No other text currently available covers this important area and uses any technological tools to enhance learning. I am confident that the economics profession welcomes a new generation Math-Econ textbook with both theoretical and computational orientation, utilizing some of the readily available computer tools for formalizing and solving many economic models that currently limited to graphic presentation. We should also note that a text of this nature will be even more attractive to new generation of students who are computationally very savvy. Each instructor will, of course, emphasize different topics covered in the book. It has been my intention to put together a book with rich ingredients to allow Math-Econ teachers to select the menu that will be closest to their students level and need.

The book is self-sufficient and self-contained. Chaps. 3 and 5 in the book cover all the mathematical tools that students need in order to understand topics covered in other chapters. It is only assumed that students have a good knowledge of algebra and have taken introductory micro- and macro-economics. Basic macro- and micro-economics are universally taken by students majoring in economics or business and in many cases by nonmajors as a part of social science core requirement.

I have attempted to connect mathematical models developed, formulated, and discussed in each chapter to real world problems by using, as much as possible, actual data in the process of operationalizing and solving the models. I consider this approach to be a distinct feature of this text.

In addition to a large number of economic examples specified and solved in each chapter, exercise sections of chapters contain a large number of problems covering application of chapter's material in various real world situations and fields of economics. As an example, exercise section of Chap. 6 that deals with various optimization problems and short- or long-run equilibrium is nine pages long. This is another desirable feature of this book.

I owe thanks to many people, including some of my students and colleagues at Fordham University, who, in one form or another, have been helpful in the process of writing this book over several years. I would like to thank all of them, especially Keith Tipple, Shanu Bajaj, Mohammed Khalil, and Yuichi Yokoyama. I would like to thank Joseph Bertino and Michael Malenbaum for proofreading, editing, and offering many helpful comments and technical assistance. I also wish to express my gratitude to Dr. Gregory Bard of the Department of Applied Mathematics and Computer Science at the University of Wisconsin-Stout, for his encouragement and valuable suggestions.

Above all, I am indebted to my wife Firoozeh and my sons Behrang and Seena for their unwavering support and care.

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