**METHODS OF FRACTURE MECHANICS:
SOLID MATTER PHYSICS**

SOLID MECHANICS AND ITS APPLICATIONS

Volume 51

Series Editor: G.M.L. GLADWELL

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Waterloo, Ontario, Canada N2L 3G1*

Aims and Scope of the Series

The fundamental questions arising in mechanics are: *Why?*, *How?*, and *How much?* The aim of this series is to provide lucid accounts written by authoritative researchers giving vision and insight in answering these questions on the subject of mechanics as it relates to solids.

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The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of the field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

For a list of related mechanics titles, see final pages.

Methods of Fracture Mechanics: Solid Matter Physics

by

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SPRINGER-SCIENCE+BUSINESS MEDIA, B.V.

A C.I.P. Catalogue record for this book is available from the Library of Congress.

ISBN 978-90-481-4794-6

ISBN 978-94-017-2262-9 (eBook)

DOI 10.1007/978-94-017-2262-9

Printed on acid-free paper

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Originally published by Kluwer Academic Publishers in 1997

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**To the memory
of my mother
Alexandra Petrovna Cherepanova**

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AUTHOR'S PREFACE

A method is a way of achieving a result. Without mastering a method, one can only hope to memorize the result. Using the result without proper knowledge of the underlying method and hence neglecting the limitations of this result usually leads to terrible mistakes. Therefore, methods form a most substantial part of a discipline. This book is for those hard-working students who desire to master the methods for the shortest time. Look at the problems placed at the end of a Chapter and try to solve them. If you can solve these problems without reading the Chapter, you do not need to scrutinize it.

Your skills in mastering the methods of the Chapter are proportional to the number of problems you can solve. This is the main indicator of a successful study of the Chapter. The highest praise of the book would be an acquired ability of the reader to solve all these problems. The methods are treated on some particular exemplar cases, which are usually simpler than the corresponding problems at the end of Chapters. To help to solve the latter, hints, sometimes in many details, are provided. So, the Chapter problems constitute an inalienable part and play a decisive role in the book. This approach allows one to get into the complicated methods in a straightforward manner, but it requires an intense work of the reader to solve each Chapter's problems. As an example of the similar approach or style, the well-known series of books on theoretical physics by Landau and Lifshitz can be mentioned. A systematic treatment of all these methods would necessitate dozens of volumes. The solutions manual is available to qualified users. The book is rich in topics for MS and PhD theses in the area of solid and fracture mechanics and physics. Therefore, it will be useful for a teacher and one who seeks a degree in this field.

Every discipline uses the known methods accommodated to its specific demands and develops its own methods that penetrate and influence other disciplines. The present volume is dedicated to some applications of the methods of fracture mechanics in solid matter physics. It is planned that the following volumes will be dedicated to some applications of these methods in mechanics of solids, theory of materials, structural integrity, material processing and manufacturing, optimal design, technology, and geophysics.

The topics are selected from the most challenging problems that are interesting for applications. As a supplementary text, the book can be used by teachers and graduate students in core courses on fracture mechanics, solid matter physics, and mechanics of solids, or in a special course on the topic of a Chapter. For convenience of reading, pertinent references are placed at the end of every Chapter. Every Chapter begins with a short preamble indicating the main methods and procedures treated in the Chapter. For those who are interested mostly in methods, it is recommended that, at first, they look through the preambles.

The basic methods used and developed in the book are as follows:

- **Mathematical modeling** methods including identification methods, the method of analogy, trial-and-error methods, and Mach's "thought economy" principle;
- **Methods of solving ordinary and partial differential equations, functional and finite difference equations of discrete mathematics, singular and regular integral equations, and integro-differential equations** which appear in various boundary value problems of fracture mechanics ;

- Methods of solving miscellaneous boundary value problems of fracture mechanics including: **asymptotic methods of boundary layer, the method of self-similar - solutions, the matched asymptotic expansions, the method of characteristics, integral transform methods, the Wiener-Hopf and Noble-Jones methods of reduction to the Riemann and Hilbert problems of the theory of functions of a complex variable, and the perturbation method;**
- Method of **invariant integrals, method of singular and generalized solutions, and method of driving or configurational forces**, which were first developed in fracture mechanics;
- **Methods of fractal geometry, catastrophe theory or singular mappings;**
- **Numerical methods** including finite element and finite difference methods.

The most important stage in the study of a problem is the formulation of a predictive model; it should properly address the main challenges of the problem. From an explorer, this stage requires the knowledge of a certain critical amount of specific basic information about the underlying phenomenon (original data base), a logical comprehensive analysis of the information (data analysis), and as a result, an intuitive finding of the mathematical model which can properly describe the main features of the phenomenon. Depending on the goals of an investigation one model can fit better than another one. Inventing and using different models make up the arts and skills of experts in any field of knowledge. In brief, a good model should be:

- Logically self-consistent, that is, not contradicting itself;
- Adequate to a corresponding real phenomenon;
- Predictive, that is, capable of predicting events and situations outside an original data basis, within the limits of necessary accuracy;
- Consistent, that is, not contradicting the approved models in a common area of their action;
- Verifiable, that is, capable of being checked by test or mind;
- Practical, that is, simple and convenient for use;
- Economical, that is, containing a necessary minimum of new constants; and last but not the least
- Beautiful, that is, delivering aesthetic emotions to the inventor and users.

As a rule, independent explorers studying the same problem invent different models, and only one of them survives. A model can be powerful or less powerful depending on the breadth and depth of its predictions. It is recognized that many models can possibly describe a phenomenon, but there is only one which is best. Creation and search for the best models is the final goal of any science or knowledge. Those who know methods better are more successful in the model invention.

Fracture mechanics has influenced the fundamentals of solid matter physics. In this book we consider the application of basic methods and ideas of fracture mechanics to the following problems of solid matter physics.

- Surface energy of solids (Chapter 1);
- Fluctuations and the kinetic theory of fracture (Chapter 2);
- Crack nucleation (Chapter 3);
- Physics of sintering (Chapter 4);

- Point defects in solids (Chapter 5);
- Dislocation emission (Chapter 6);
- Relativistic electron beams in a solid (Chapter 7);
- Fractals in fracture of solids (Chapter 8).

More detailed information about the Chapter is given in Table of Contents.

Many people have, directly or indirectly, been involved in the creation of the book.

My wife Larisa Cherepanov typed most part of the book and prepared figures. Without her devotion to the project, the book could not have been written. My senior colleagues and former friends, first of all Lev Galin, Vadim Sokolovsky, Grigory Barenblatt and Leonid Sedov, influenced my way of thinking and style of work. Jock Eshelby, John Gilman, Jim Rice, Bernie Budiansky, John Hutchinson and others offered constructive criticism of some topics of the book. It is my duty to express my sincere gratitude to all of them .

My special thanks go to Professor Gladwell, Editor of this series, who generously read a draft of the entire manuscript. The help of Helen Rooney, Cathy Rustad, Maggie Zabalo and Caroline Liberty, is gratefully acknowledged. Many other individuals involved in the project are identified on the relevant pages of the book or in the bibliography.

Some results of investigations treated in the book were found thanks to NSF Grant MSS 9224936, NASA Grant 541803700, AFOSR Grant 59107, and U. S. Army SSDC Grant DASG 60-94-C-0032 funding author's research during 1991-1995.