Chapter Four

LAMINATE THEORIES

Although classical lamination theory has been the "workhouse" model for composite laminate analysis, the large errors in its deflection prediction at low span–to–depth ratios (*i.e.*, thick plates) suggest the need for refinements in that regime. With that in mind, Mindlin–Reissner type theories were developed by Yang, Norris, and Stavsky[1] and J. M. Whitney and myself[2] under slightly different assumptions. Aside from more precise deflection predictions, vibration frequencies and buckling loads were also considered. Actually, the stresses were unaffected by the assumption of shear deformation in the class of boundary value problems considered.

Another chapter in the book edited by Sendeckyj raised the issue of effective laminate properties, *i.e.*, in what sense can a laminate be regarded as a homogeneous body? This chapter has apparently gone unnoticed for a long time as several, less general, models have been developed in recent years. What is presented here is actually an exact elasticity solution for a body which possesses arbitrary heterogeneity in the thickness direction – lamination of anisotropic layers representing a special case. The solution of the model is carried out in such a way as to provide "constitutive" equations, or relations between imposed forces and moments per unit length and the respective mechanical and non–mechanical strains and curvatures. For the usual class of laminates built from monoclinic layers, the relations reduce to hose of classical lamination theory, thus demonstrating that the latter *is an xact solution* (if the layers are truly homogeneous) in regions away from edges ıd other forms of stress concentration.

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[1] P. C. Yang, C. H. Norris, and Y. Stavsky, "Elastic Wave Propagation in ɛterogeneous Plates," *Int. J. Solids and Structures*, Vol. 2, 665–684 (1966).

J. M. Whitney and N. J. Pagano, "Shear Deformation in Heterogeneous sotropic Plates", *J. Applied Mechanics*, Vol. 37, No. 4, 1031–1036 (1970).