

MECHANICS OF COMPOSITE MATERIALS

SOLID MECHANICS AND ITS APPLICATIONS

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Mechanics of Composite Materials

Selected Works of Nicholas J. Pagano

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PREFACE

Composite materials is currently the hottest topic of researchers in chemistry, chemical engineering, electrical engineering, material science, mechanical engineering, and solid and structural mechanics. Composite materials are used in a variety of engineering structures, including aerospace, automotive, and underwater structures, as well as in medical prosthetic devices, electronic circuit boards, and sports equipment. In the last three decades, there has been a major effort to develop composite material systems, and analyze and design structural components made from composite materials.

Everyone involved with the mechanics of composite materials and structures must have come across the works of Dr. N. J. Pagano in their research. His research papers are among the most referenced of all existing literature in the field of mechanics of composite laminates. This monograph has the objective of making available, in one volume, all major technical papers of Dr. Pagano.

There are eight general topics treated in this monograph. They are arranged approximately in chronological order, although there is some overlapping. It was felt that the topical organization is more important than the historical consistency. Most of the papers are published in the open literature, but there are a few exceptions—a few key unpublished reports have been included for continuity. The topics are:

1. Some basic studies of anisotropic behavior
 - (a) testing
 - (b) theoretical concerns
 - (c) curing stresses
2. Exact solutions for elastic response
 - (a) laminates
 - (b) cylinders
3. Role of micromechanics

4. Lamination theories
5. Interlaminar stresses
6. Involute (Rosette) bodies
7. Some carbon-carbon spinoffs
8. Micromechanics of BMC

The technical papers of Dr. Pagano are arranged in the order of the above topics. Dr. Pagano wrote some personal and historical reflections and suggested topics of future research activity. A brief overview of Dr. Pagano's technical contributions is presented next.

Dr. Pagano has made many pioneering contributions to the mechanics of composite materials and structures. In particular, he has contributed significantly to *delamination of composite laminates*, *analysis of involute construction*, and *failure models for brittle matrix composites*.

Dr. Pagano's classic work on delamination of composites explains the mechanism controlling free-edge delamination and it is the starting point for researchers in the field. He has made major contributions to all three key issues of delamination, namely, a fundamental understanding of the failure mode, formulation of response models, and development of test methods to characterize the phenomenon.

Dr. Pagano's pioneering and rigorous work that describes the geometry and elastic response of involute bodies, a complex form of laminated composite, is used by all analysts working with rocket motor involute structures. His patented exact involute approach and theoretical methods were transitioned to industry with amazing speed through close-working informal arrangements with industrial and government engineers in the United States and this led to drastic changes in the practices used by the industry.

Dr. Pagano currently leads an in-house research program in the development of analytical models to describe the thermomechanical response of composite materials in the presence of phase material damage including the fiber-matrix interface. Such models are relevant to damage tolerance issues in weapon systems and aerospace vehicles exposed to very high temperatures (such as 2,500 °F) and which contain components built from ceramic- and glass-ceramic-matrix composite materials.

Dr. Pagano published many high quality research papers, as evidenced by this monograph. He is also the author/editor of three books: *Elasticity, Tensor, Dyadic, and Engineering Approaches*, (with P. C. Chou), Van Nostrand (1967), *Composites Materials Workshop*, (with S. W. Tsai and J. C. Halpin) Technomic Publishers (1968), and *Interlaminar Stresses in Composite Materials*, Elsevier (1988) (translated into Russian by Mir Publishing Co., 1991).

Dr. Nicholas J. Pagano is currently a senior scientist in Mechanics & Surface Interactions Branch, Nonmetallic Materials Division, Materials Directorate, Wright Laboratory (WL/MLBM), Dayton, Ohio.

The editor is very thankful to Dr. Pagano for allowing him to have the distinct privilege and honor of editing this monograph. It is very satisfying to see most major technical works of Dr. Pagano in one volume. Due to space limitation, several others works of Dr. Pagano (especially, papers cited in Chapters 6 and 8) were not included in this volume. It is hoped that the monograph will be well-received by the mechanics community.

J. N. Reddy
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ABOUT THE EDITOR

Professor J. N. Reddy is the inaugural holder of the *Oscar S. Wyatt Endowed Chair* in Mechanical Engineering at Texas A & M University, College Station, Texas. Dr. Reddy authored numerous papers on the theory and finite element analysis of problems in continuum mechanics, solid and structural mechanics, laminated composite plates and shells, computational fluid mechanics, numerical heat transfer, and applied mathematics. Dr. Reddy is the author and coauthor of eight text books, including: *An Introduction to the Finite Element Method*, McGraw-Hill, 1994; *Energy and Variational Methods in Applied Mechanics*, John Wiley & Sons, 1984; and *Applied Functional Analysis and Variational Methods in Engineering*, McGraw-Hill, 1986. Dr. Reddy serves on the editorial boards of numerous journals, including *International Journal for Numerical Methods in Engineering*, *International Journal for Numerical Methods in Fluids*, *Journal of Applied Mechanics*, and he is the Editor-in-Chief of *Mechanics of Composite Materials and Structures*.