Game-Theoretic Methods in General Equilibrium Analysis

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Game-Theoretic Methods in General Equilibrium Analysis

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