A PRIMER IN ELASTICITY

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by

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A Sara, Marta e Adriano

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Preface

Linear Elasticity is “the mother of all theories” in continuum physics. In the century now about to close its traditional devotees – the mathematician and the engineer – have been joined by geophysicists, biomechanists, materials scientists, and other users. By a seeming paradox, the variety and disparity of current applications calls for a study of elasticity *per se*. This book, which is directed to both mathematicians and physical scientists, presents the foundational issues of linear elasticity in a compact, unabridged manner. Since its intended readership certainly includes that of the *Journal of Elasticity*, I have welcomed the opportunity I have been offered by Kluwer to reach that primary target by publishing this writing first in that journal, under form of a special presentation invited by the Editor-in-Chief, Professor R.L. Fosdick.

There are four chapters: the first two illustrate, respectively, the concepts of deformation and strain and of force and stress; the third is devoted to a study of constitutive relations; the last discusses the posing of equilibrium problems. The emphasis is in the description of elasticity as a model whose construction calls for a delicate interplay between physics and mathematics. The conceptual links with general continuum mechanics – in particular, with finite elasticity – are carefully indicated, especially when a notion, as is the case with strain and stress, is better understood when presented in full. As is to be expected when the goal is to discuss the foundations of a theory in a slim essay, no pretenses of comprehensiveness are made, but occasional concessions to personal taste are found.

This book could not be written without the knowledge gained from two masterful works by M.E. Gurtin, his Handbuch article on linear elasticity of 1972 and his *Introduction to Continuum Mechanics* of 1981. Its peculiarities are an updated presentation of the basic concepts and an unusual coverage: it would not be easy to find in one other book in elasticity a treatment of such issues as exact and linearized equilibria, the constitutive problems of classification and representation, internal constraints and material symmetries, elastic equilibrium with the Cauchy relations, and elastic equilibrium in the presence of internal constraints. As the title suggests, this is meant to be a short, condensed handbook to be used to teach. In fact, the book is an outgrowth of advanced undergraduate and graduate courses I have repeatedly given, in my own institution and elsewhere, for students in applied mathematics and engineering; for this reason, a score of figures and one hundred exercises of variable difficulty are included.

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P. PODIO-GUIDUGLI
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