Stochastic Processes and Operator Calculus on Quantum Groups

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Stochastic Processes and Operator Calculus on Quantum Groups

by

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Preface

Quantum groups have been investigated rather deeply in mathematical physics over the last decade. Among the most prominent contributions in this area let us mention the works of V.G. Drinfeld, S.L. Woronowicz, S. Majid. Probability theory on quantum groups has developed in several directions (see works of P. Biane, R.L. Hudson and K.R. Partasarathy, P.A. Meyer, M. Schürmann, D. Voiculescu). The aim of this book is to present several new aspects related to quantum groups: operator calculus, dual representations, stochastic processes and diffusions, Appell polynomials and systems in connection with evolution equations. Much of the material is scattered throughout available literature, however, we have nowhere found in accessible form all of this material collected. The presentation of representation theory in connection with Appell systems is original with the authors. Stochastic processes (example: Brownian motion, diffusion processes, Lévy processes) are investigated and several examples are presented.

As a text the work is intended to be accessible to graduate students and researchers not specialised in quantum probability.

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