

Numerical Methods for the Solution of Ill-Posed Problems

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Numerical Methods for the Solution of Ill-Posed Problems

by

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Preface to the English edition

The Russian original of the present book was published in Russia in 1990 and can nowadays be considered as a classical monograph on methods for solving ill-posed problems. Next to theoretical material, the book contains a FORTRAN program library which enables readers interested in practical applications to immediately turn to the processing of experimental data, without the need to do programming work themselves. In the book we consider linear ill-posed problems with or without a priori constraints. We have chosen Tikhonov's variational approach with choice of regularization parameter and the generalized discrepancy principle as the basic regularization methods. We have only fragmentarily considered generalizations to the nonlinear case, while we have not paid sufficient attention to the nowadays popular iterative regularization algorithms. A reader interested in these aspects we recommend the monograph: 'Nonlinear Ill-posed Problems' by A.N. Tikhonov, A.S. Leonov, A.G. Yagola (whose English translation will be published by Chapman & Hall) and 'Iterative methods for solving ill-posed problems' by A.B. Bakushinsky and A.V. Goncharsky (whose English translation has been published by Kluwer Acad. Publ. in 1994 as 'Ill-posed problems: Theory and applications'). To guide the readers to new publications concerning ill-posed problems, for this edition of our book we have prepared a Postscript, in which we have tried to list the most important monographs which, for obvious reasons, have not been included as references in the Russian edition. We have not striven for completeness in this list.

In October 1993 our teacher, and one of the greatest mathematicians of the XX century, Andreï Nikolaevich Tikhonov died. So, this publication is an expression of the deepest respect to the memory of the groundlayer of the theory of ill-posed problems. We thank Kluwer Academic Publishers for realizing this publication.

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