

# ELEMENTS OF STRUCTURAL OPTIMIZATION

# SOLID MECHANICS AND ITS APPLICATIONS

## Volume 1

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# Elements of Structural Optimization

by

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*This book is dedicated to*

**Rose**

**Nihal, Tacettin, and Pinar**

**Evelyne, Subhash, and Nikhil**

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The field of structural optimization is still a relatively new field undergoing rapid changes in methods and focus. Until recently there was a severe imbalance between the enormous amount of literature on the subject, and the paucity of applications to practical design problems. This imbalance is being gradually redressed now. There is still no shortage of new publications, but there are also exciting applications of the methods of structural optimizations in the automotive, aerospace, civil engineering, machine design and other engineering fields. As a result of the growing pace of applications, research into structural optimization methods is increasingly driven by real-life problems.

Most engineers who design structures employ complex general-purpose software packages for structural analysis. Often they do not have any access to the source program, and even more frequently they have only scant knowledge of the details of the structural analysis algorithms used in this software packages. Therefore the major challenge faced by researchers in structural optimization is to develop methods that are suitable for use with such software packages. Another major challenge is the high computational cost associated with the analysis of many complex real-life problems. In many cases the engineer who has the task of designing a structure cannot afford to analyze it more than a handful of times.

This environment motivates a focus on optimization techniques that call for minimal interference with the structural analysis package, and require only a small number of structural analysis runs. A class of techniques of this type, pioneered by Lucien Schmit, and which are becoming widely used, are referred to in this book as sequential approximate optimization techniques. These techniques use the analysis package for the purpose of constructing an approximation to the structural design problem,

## *Preface*

and then employ various mathematical optimization techniques to solve the approximate problem. The optimum of the approximate problem is then used as a basis for performing one or more structural analyses for the purpose of updating or refining the approximate design problem. Most of the approximate design problems are based on derivatives of the structural response with respect to design parameters.

In the new environment the structural designer is typically called upon to provide the interface between a commercially available analysis program, and a commercially available optimization software package. The three most important ingredients of the interface are: sensitivity derivative calculation, construction of an approximate problem, and evaluation of results for the purpose of fine-tuning the approximate problem or the optimization method for maximum efficiency and reliability.

This textbook is organized so that its middle part—Chapters 6, 7 and 8 deal with the two issues of constructing the approximate problem and obtaining sensitivity derivatives. Evaluating the results of the optimization calls for a basic understanding of optimality conditions and optimization methods. This is dealt with in Chapters 1 through 5. The last three chapters deal with the specialized topics of optimality criteria methods, multi-level optimization, and applications to composite materials.

The material in the textbook can be used in various ways in teaching a graduate course in structural optimization, depending on the available amount of time, and whether students have prior preparation in optimization techniques.

Without prior preparation in optimization techniques it is suggested that the minimum time requirement is one semester. It is suggested to cover Chapter 1, sections 2.1, 2.2 and 2.3 of Chapter 2, Sections 3.1 and 3.4 of Chapter 3, some material from Chapters 4 and 5 depending on the instructor's favorite optimization methods, most of Chapter 6 and the first two sections of Chapter 7. With a two-quarter sequence it is suggested to cover Chapters 1 and 2, selected topics of Chapters 3 to 5 and Chapter 6 in the first quarter, and Chapters 7, 9, 11 and either Chapter 8 or Chapter 10 in the second quarter. Finally, in a two-semester sequence it is recommended to cover Chapters 1 through 6 in the first semester, and Chapters 7 through 11 in the second semester.

With a preparatory course in mathematical optimization a one quarter and a one semester versions of the course can be considered. A one-quarter version could include Chapters 1 and 2, sections 3.1, 3.2, 3.3 and 3.7 of Chapter 3, and Chapters 6, the first two sections of Chapter 7, and Chapter 9 or 11. A one-semester version could include the same part of Chapters 1 through 7 and then Chapters 9 through 11.

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