Limit Theorems for Random Fields with Singular Spectrum

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Limit Theorems for Random Fields with Singular Spectrum

by

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Preface

This book is devoted to an investigation of the basic problems of the theory of random fields which are characterized by certain singular properties (e. g., unboundedness, or vanishing) of their spectral densities. These random fields are called, the random fields with singular spectrum, long-memory fields, random fields with long-range dependence, fields with slowly decaying correlations or strongly dependent random fields by various authors. This phenomenon has been observed empirically by many scientists long before suitable mathematical models were known. The methods and results differ significantly from the theory of weakly dependent random fields.

The first chapter presents basic concepts of the spectral theory of random fields, some examples of random processes and fields with singular spectrum, Tauberian and Abelian theorems for the covariance function of singular random fields.

In the second chapter limit theorems for non-linear functionals of random fields with singular spectrum are proved.

Chapter 3 summarizes some limit theorems for geometric functionals of random fields with long-range dependence.

Limit distributions of the solutions of Burgers equation with random data via parabolic and hyperbolic rescaling are presented in chapter 4.

And chapter 5 presents some problems of statistical analysis of random fields with singular spectrum.

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