

VORTEX PROCESSES AND SOLID BODY DYNAMICS

FLUID MECHANICS AND ITS APPLICATIONS

Volume 25

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Vortex Processes and Solid Body Dynamics

The Dynamic Problems of
Spacecrafts and Magnetic Levitation Systems

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Preface

*...a wise man knows all things in a manner
in which this is possible, not, however,
knowing them individually.*

Aristotle. *Metaphysics**

The problem of consideration of vortex fields' influence on solid body dynamics has a long history. One constantly comes upon it in flight dynamics of airplanes, helicopters, and other flying vehicles (FV) moving in the atmosphere, in dynamics of ships with hydrofoils, and in dynamics of rocket carriers (RC) and spacecrafts (SC) with liquid-propellant rocket engines (LPRE), that are equipped with special damping devices and other structural elements inside fluid tanks.

Similar problems occur when solving problems related to attitude control and stabilization of artificial Earth satellites (AES) and spacecrafts with magnetic (electromagnetic) systems, in conducting elements of which eddy currents are induced while control of those vehicles' angular position. It is also true with special test facilities for dynamic testing of space vehicles and their systems, with modern high-speed magnetic suspension transport systems (those based on the phenomenon of 'magnetic levitation'), with generators having rotors carried in 'magnetic bearings', and so on.

At certain stages of research it was possible to assume, that vortex fields were localized within a thin boundary layer, and to consider them a by-effect. Recently, however, due to rapid complication of both vehicles themselves and their control systems (systems of stabilization and attitude control), still sharper has become the problem of inclusion into the vehicle's mathematical model of one or another model of 'non-small' vortex fields, the kinetic energy of which is comparable to potential fields kinetic energy. It is true for low-viscous liquid vortex motions caused by various stalling effects, as well as for magnetic fields induced by eddy currents in cores having comparatively high electrical conductivity. The problem of control of those fields moves to the foreground.

Complete mathematical description of unsteady vortex fields requires the introduction of a number of hardly formalizable physical factors and solution of complicated boundary-value problems (especially with stalled flow). These questions are currently covered with the most completeness in the literature on winged vehicle dynamics and on aeroelasticity, with respect to the problems of unsteady flow around wings, tail units, and the flying vehicle as a whole (external boundary-value problems).

Literature devoted to dynamics of rocket carriers and spacecrafts with liquid-propellant rocket engines having tanks equipped with liquid sloshing dampers and

* Transl. with comment. and glossary by Hippocrates G. Apostle. Bloomington—London: Indiana University Press, Cop. 1966, (p. 14)

literature on eddy currents in cores reduce analysis of this problem to investigation of the influence of purely dissipative effects (these are mainly internal boundary-value problems), that, naturally, narrows the statement and the investigation of the problem.

The present book is devoted to a more complete investigation of that class of problems, including not only dissipation effects, but also the effect of liquid vortex motion kinetic energy. Dynamics of objects, for which vortex fields may play a dominant role, is described based on a common phenomenologic model of unsteady eddy currents and vortex motion of liquid.

The appropriate mathematical models describing the considered objects' motion are sets of singular integrodifferential equations allowing fruitful investigation with modern analytical and numerical methods. Phenomenologic models of that kind let us obtain information of the system's behaviour as a whole ("know all things in a manner in which this is possible") and reveal a number of new delicate dynamic effects, not knowing the details of eddies distribution ("not, however, knowing them individually").

The book consists of eight chapters. Chapters 1 and 2 are devoted to the synthesis of phenomenologic mathematical models of objects, for which the consideration of vortex fields plays a dominant role in the formulation of those models.

Success of those models application to the analysis of complex controlled systems dynamics is determined by the possibility of creating, based on the models, efficient analytical and numerical algorithms for perturbed motion investigation and for solution of the system 'object – regulator' stability problems. Note, that though obtained mathematical models seem simple, the general solution of the sets of integrodifferential equations, that the models yield, is by no means trivial. Chapter 3 deals with this question.

Chapter 4 is devoted to the experimental checking of the introduced models and to estimation of their applicability limits.

The first four chapters form thus a basis for an unbiased estimation of the influence of new physical factors, reflected in the suggested mathematical models, on dynamics of the stabilization and attitude control systems elements and on dynamics of the closed system 'object – regulator'.

Chapter 5 analyzes influence on the electromagnetic levitation system stability of eddy currents in ferromagnetic core elements having comparably high electrical conductivity (HECF *elements*). Electromagnets with controlled winding currents are the actuators of that system. Chapter 6 presents an analysis of the influence on spacecrafts dynamics and stability of vortex motions of the low-viscous liquid (*LV liquid*) partially filling the vehicle's tanks and characterized by high vorticity caused by damping ribs.

Those two chapters successively consider mathematical models of various degrees of completeness, including traditional models, that take into account only dissipative components of the adjacent vortex fields. Some model problems are considered, which solutions enable an estimation of those factors' influence on the 'object – regulator' closed system dynamics. Examples within the chapters are intended to illustrate the possibility of proceeding from "knowing individual things" to "the general view on

similar things” and the possibility to allow for different physical phenomena in a common formalism. On the other side, those examples let you judge, in which cases and to what extent it is necessary to use the suggested more exact mathematical models, and when the simplified ones are sufficient.

Chapter 7 presents examples of synthesizing a control law based on one of the models of Chapter 1 for the air gap stabilization of the magnetic levitation system of the test facility.

Somewhat separate is Chapter 8 completing the book. It deals with the synthesis and investigation of a general mathematical model of the system ‘solid – low-viscous electrically conductive ferromagnetic liquid (LVECF *liquid*)’, which is based on the classical equations of magnetohydrodynamics [11–13]. In particular cases low-viscous conducting liquid (LVEC *liquid*) may be an element of that system, the magnetic field being implemented with a special coil in a body related coordinate system. The mathematical models of the first two chapters may be obtained from that model as particular cases.

All the problems considered in those eight chapters may be related to non-classical problems of solid body dynamics, where vortex fields of one or another physical nature play a dominant role.

Chapters 1 and 2 are written by Boris Rabinovich; Chapter 3 by Valery Lebedev, Alexander Mytarev, and Boris Rabinovich together; Chapters 4 and 6 by Valery Lebedev together with Alexander Mytarev; Chapters 5 and 7 by Alexander Mytarev, Chapter 8 by Boris Rabinovich together with Alexander Mytarev.

The authors would like to express their acknowledgement to Professor V. Beletsky for a number of useful advices when reviewing the manuscript and to A.S.Leviant, Candidate of Science in Physics and Mathematics, for translating the book into English.

The authors.

Preface to the English Edition

The present edition is an English translation of the authors' book having the same title (published in Moscow by Nauka Publishers, the Main Editorial Board for Physical and Mathematical Literature, in 1992).

This book differs from the mentioned one in essential extension of Chapter 8, to which three new articles are added. Those articles are devoted to revealing dynamic features of the system 'solid body – electrically conductive liquid moving in magnetic field related to the body.'

The list of bibliography includes, where available, references to the English language editions of papers and books published originally in Russian. Besides that, some denominations are changed in order to make them closer to those traditional for the English language scientific literature. The noticed misprints were corrected.

The authors

List of Main Symbols

1. Constants and Functions of One or Several Arguments

A, Ψ, a, β – scalars; $\mathbf{R}, \vec{\phi}, \mathbf{r}, \vec{\alpha}$ – vectors; \mathbf{J}, \mathbf{A} – tensors and matrices; \mathfrak{S} – operators;
Sh – dimensionless congruence criteria.

2. Superscripts

$\dot{}$ – time derivative;
 $\prime, \prime\prime$ – coordinate derivatives; quantities associated with the fixed cover model of liquid motions; in particular places of the book they denote some coefficients at the first and second derivatives and some auxiliary coefficients;
 $\bar{}$ – conjugated quantity (a tensor or a complex quantity);
 \sim – normalized quantity, particularly a dimensionless one;
 \wedge – boundary layer type function; a temporary symbol in intermediate transformations;
 $^\circ$ – quantity related to unperturbed motion or state (in the latter case it means absence of perturbations associated with air gap, liquid fluidity, presence of damping ribs inside a tank, etc.);
 $^{(0)}$ – quantity related to solidified liquid;
 $*$ – quantity directly associated with vortex motions of liquid or with eddy currents;
 e – external domain;
 i – internal domain;
 a – self-sustained oscillation processes in the roll channel.

3. Subscripts

F – quantity characterizing liquid motion;
 M – quantity associated with electromagnetic field;
 E – experimental value;
 O, G – the point, the given quantity is referred to, in particular, it is the system's centre of mass;
 0 – initial value in Cauchy problem or initial estimate;
 a – absolute motion of liquid; self-sustained oscillations.

4. Adopted Abbreviations

HV liquid – high-viscous liquid;
LV liquid – low-viscous liquid;
LVEC liquid – low-viscous electrically conductive liquid;
LVECF liquid – low-viscous electrically conductive ferromagnetic liquid;
HECF element – ferromagnetic element with high electrical conductivity;
LECF element – ferromagnetic element with low electrical conductivity;
LPR engine – liquid-propellant rocket engine.