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CARDINALISM

A Fundamental Approach

Edited by

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and

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FOREWORD

THE CONCEPTION OF THIS VOLUME

This volume, *Cardinalism*, has been initiated by Ole Hagen, and is now published due to his perseverance and to Kluwer Academic Publishers.

Because of various activities and duties, my contribution to the general conception of this volume has only been formal, and all the credit for it is due to Ole Hagen. I should also emphasize that the responsibility for the year's delay in the publishing of this volume is entirely mine, for two reasons. First of all, I have been involved in many works in very different fields. Second, the English translation of my 1943 contribution to the concept of cardinal utility took some time.

The points of view the reader will find in this volume are often different and sometimes contradictory, but this can only increase the interest of its reading. In any case, this is not the editors' part to side with or against. Thus contributions to this volume are presented as they have been submitted to the editors. Of course, this does not mean that they entirely agree with the analyses presented.

MAURICE ALLAIS

INTRODUCTION

Everyone's conscious choices are assumed to reflect their preferences in different situations. For some purposes it serves the theorist's preference for simplicity to assume that a person's life style can be described by the values of some concrete variables, such as quantities of goods available.

We must make allowance for the philosophical possibility that as well as preference there is also indifference, but we do not believe that a human or even an ass would, like that of Johannes Buridan, die from hunger between two equally attractive haystacks. We further assume transition between the preferences.

Now if we attach numbers to each situation in inverse order of preferences, we have what in the literature is generally named an ordinal utility function. If we subject all the numbers to the same strictly positive transformation, we still have an ordinal utility function. But why is it numbered in inverse order, and why is it called utility? Like pure paper money, this can only be explained historically. We shall return to this problem.

The order of the set of all preferences can be divided into subsets defined in such a way that any element within a subset has numbers that are positive linear transformations of those of any other element in that subset.

If a person claims a particular validity or relevance or usefulness for one of these sets, he or she is a cardinalist and has defined (the observed person's) alleged cardinal utility up to a positive linear transformation. Of course a cardinal utility is also an ordinal utility. So what is an ordinalist? It is a person who believes that no cardinal utility can exist, or if it does the fact is totally irrelevant and of no consequence.

We know that cardinalism is older than ordinalism, but we do not know how old cardinalism is. It is of no use to search for its beginnings among the classics of economics. As an example: About 1900 years ago a story was written about an offering in a temple. People donated gifts of various magnitudes. Up came a poor widow who gave (in free translation) two pennies. A lay preacher, son of a carpenter, who was in the habit of making surprising statements, said that she gave more than the others, because they gave from their abundance, while she gave her means of subsistence. She "gave more". More of what? Though the term was not coined, what was meant could only be a piece of cardinal utility.

The originator of this story was not only a cardinalist, he also foreshadowed the theory of the diminishing utility of money, and took sides in the discussion on interpersonal comparison, in which there are today cardinalists on both sides.

When economic theorists began to focus their interest on the demand side of the market and specifically consumer's choices, the notion of utility (later called cardinal –) evolved quite naturally like that of the ancient storyteller (who probably was not the first). It was just a short step then to Gossen's law; of course if the amount of utility could be increased by shifting a penny from buying one good to buying another, this would be done, so marginal utility/price must be the same for all goods in the point of optimal budget allocation.

Pareto stated (1911) that the map of a consumer's (one period) preferences was invariant under a strictly positive transformation, following from Gossen's law.

It was not till the 1930s that significant conclusions were made that marked the beginning of the ordinalistic revolution. Cardinal utility became taboo. The non-belief in the existence of cardinal utility became a firm, devout belief in its non-existence.

At this stage let it be admitted that any preferences ordering can of course be *described* by ordinalistic theory, but can it be *explained*? And can from the base of ordinalism hypotheses be formed without, maybe subconsciously drawing on the deductions that follow from the cardinalistic notions? The chart of indifference curves, convex to the origin, which is acceptable to cardinalists and ordinalists alike – has it ever really been constructed by asking people about their preferences? Is it not really deduced from the notion of declining marginal utility ('meaningless' to the ordinalist) and then confirmed by its testable implications? And why do they name their preference function 'utility', if not because it is a cardinalistic legacy, which they have afterwards stripped of 'unnecessary' characteristics?

Is not ordinalistic theory a special case of the self-evident truth that people do what they do because they would rather do what they do than anything else they could have done. To this writer ordinalistic theory gives a feeling of *déjà vu*. What is relived is a text giving sex information to children. In both cases the reader may wonder: "Now I know what they do, but why do they do it?" What is left out in both cases are the forces that motivate actions. A force can be strong or weak. So a preference can have more or less strength behind it. In which case you have cardinality.

During half a century of ordinalistic dominance, there was a moment of embarrassment: the appearance of the von Neuman/Morgenstern utility index, which was defined up to a linear positive transformation, just as the classical cardinal utility. The problem of coexistence was "solved" by subtle and sophisticated interpretations: The index does not show the outcome, but was only a mathematical device for indirectly expressing a person's attitude to risk. Also, the index only showed the preferences between games, not the comparative strength of the preference, disregarding fact that if three games has three different values of the vN–M index,

the ratio between the two intervals can not be changed.

To the question (Does it exist?) concerning the $vN-M$ index and cardinal utility in the classical sense respectively, there are four pairs of answers actually given: YY, YN, NY and NN. In the case of YY there are two answers to the question if they are identical.

Most of the problems mentioned above and some others are discussed by the contributors to this book. It is not necessarily a balanced picture. The contributors are selected mainly in order to show what the cardinalist side has to offer. But once a contributor is invited, there has been no censorship. Unfortunately not all invitations have resulted in a contribution, for various reasons. Before proceeding any further, an introductory survey may be in order.

Maurice Allais is Professor of Economics (emeritus) at Ecole National Superier des Mines de Paris. Originally educated as an engineer, he started his academic career in 1943 by publishing *A la Recherche d'une Discipline Economique*. This book set in train a development in basic economic theory that resulted in several Nobel Prizes, the latest (1988) awarded to himself.

At that time he was in fact better known as a critic of the expected utility model for decisions under risk. His work in this field was considered very controversial and the sad truth for the academic community was that it was not taken quite seriously. The "Allais Paradox" was frequently referred to, but the comments often showed that the commentator had not bothered to study the original, with the foundations of an alternative model, even after his first publication in English in this field (1979).

Allais has from the beginning in 1982 been a pillar of strength in the series of FUR conferences (International conferences on the Foundations of Utility and Risk theory).

One chapter from Allais's book in 1943, *Satisfactions Absolues*, now appears for the first time in English, *Absolute Satisfaction*. Here he establishes cardinalism not on introspection alone, but on systematic use of experimental results and analogies from psychology and physics. In some cases he discovers old treasures, but I suspect that young scholars may have some pleasant surprises, like finding among "old hats" stored as outmoded some that are now again quite *en vogue*.

Allais's other contributions, *Cardinal Utility: History, Empirical Findings, and Applications – An Overview* and *Determination of Cardinal Utility According to an Intrinsic Invariant Model*, give an overview of the history of cardinal utility, and presents the development of his intrinsic invariant model for determining the cardinal utility of changes in wealth. It includes extensive experimental evidence. Allais's method is to ask respondents directly about utility intervals. The validity of these is tested through comparison over time. The essence of his conclusion is that

each person has a cardinal utility function, formed by a common generating function and an individual parameter. The utility functions have common characteristics that they are mostly logarithmic, but flatten out at high levels, near satiation. Individual differences are essentially caused by differences in the respondent's actual wealth.

This is the first time in the history of utility that exact measurements of utility have been carried out. The consistency over time and persons give them scientific validity.

The historical overview contains a concise presentation of Allais's whole philosophy of utility in five points.

Tore Ellingsen is in the final stage of his Ph.D at the London School of Economics and Political Science.*

In *Cardinal Utility – A History of Hedonimetry* he gives an informative and thought provoking, survey of the development of theory and also empirical work in the field. He makes no secret of his own views, and his points are mostly well taken. His presentation of the different scales is very concise, informative and clarifying, including a few scales that are often left out in short surveys.

Erik Grønn is an Associate Professor at the Norwegian School of Management. He studied and for some time worked at Frisch and Haavelmoes Institute of Economics at the University of Oslo.

In *Cardinalism and Dynamic Analysis in Economic Theory* he discusses the possible difference in the relevance of cardinality in static and dynamic analysis. Though he does not reach a final conclusion to this, he does note that ordinal utility is incapable of treating calendar time dependent preference and Boehm–Bawerk's rate of pure time preference.

Ole Hagen is Professor of Managerial Economics at the Norwegian School of Management (Oslo region), which he joined after leaving The Norwegian School of Economics and Business Administration (Bergen), after he started critical analysis of the EU theory in 1969. He has produced several articles in this field and edited books, previously jointly with Allais, and has been active in organizing the FUR conferences.

His special angle on the question of cardinality is the relevance and operationality of indirect second order derivatives. In *The Short Step from Ordinal to Cardinal Utility* he shows that second derivatives keep their interval ratios in all ordinal utility functions. What is needed for identifying the relevant cardinal subset is the distinction between positive/negative. Examples are given to support the operationality.

Manfred Kraft is Assistant Professor of Statistics, Econometrics and Decision Theory at the University of Paderborn. He has published works on statistics, applied econometrics and business cycle theory. Peter Weise is Professor of Economics at the University of Kassel. He has published works on microeconomics, labour economics and law in relation to economics.

They give an example of application of cardinal utility in macroeconomics in their study of the alterations in strength and direction in the effect of the business climate on individual decisions in *A Cardinal Utility Approach to the Theory of the Business Cycle*.

Roman Krzystofowicz is a Professor in the Department of Systems Engineering, University of Virginia. For many years he has worked with the problems of decisions under uncertainty. He has made a large scientific contribution, both in basic and applied fields, and has been the head of several large prospects. He has received three distinguished awards.

In his contributions to this book he deals with the role of cardinal utility in the context of decisions under risk.

In *Generic Utility: Explanatory Model, Behavioral Hypotheses, Empirical evidence* he gives a thorough general analysis of the matter and presents a model which decomposes the utility of a game into two factors: a value function and a risk function. He is aiming at crystallizing empirical foundations for “nonlinear” (not EU) models and filtering probability induced biases in measuring utility for EU models.

It is by now well known that attempts to determine the vN–M utility index give conflicting results depending on, for example, the probability distributions used. In *Filtering Risk Effect in Standard-gamble Utility Measurement* Krzystofowicz proposes a method characterised by using certainty equivalents of sequential 50–50 games and a simple parameter for risk attitude. There is no final conclusion. Further research is in progress.

I cannot finish this introduction without apologizing to all concerned for unforeseen delays, due in part to health reasons, which also prevented me from attending the fourth International Conference on the Foundation and Application of Utility and Risk Theory, FUR, in Budapest, 1988.

The contributors to this book have expressed various and in part conflicting views. Speaking as one of them, I think we all hope that many readers will find them provocative.**

P.T. University of Bonn in May 1990

OLE HAGEN

NOTES

* Finished 1991. He is now at the Stockholm School of Economics.

** A forum for comment could be FUR VII in Sandvika, 30 June to 3 July, 1994.

POSTSCRIPT

At the final stage of my editing part of Allais' foreword a new, original contribution by Maurice Allais was developed and included as 'The Fundamental Cardinalist Approach and Its Prospects'. It is put at the end of the volume as Chapter X.

This rounds off his own contribution as well as the structure of this book and I think it gives significant pointers for future research in the fields where cardinal utility is highly relevant.

*Norwegian School of Management
January 1994*

O.H.